

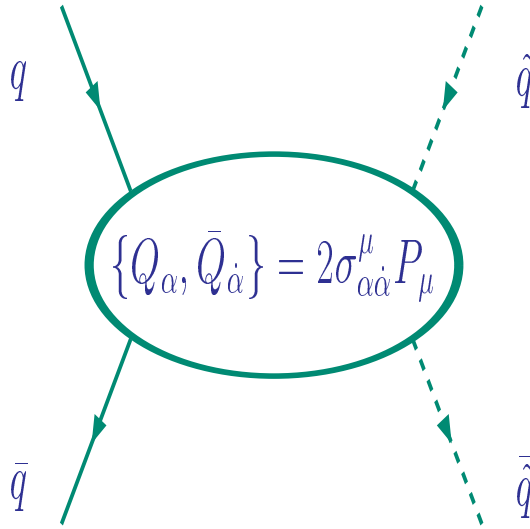
BEYOND THE STANDARD MODEL*

(IN SEARCH OF SUPERSYMMETRY)

D. I. Kazakov

BLTP, JINR, Dubna and ITEP, Moscow

e-mail: kazakovd@thsun1.jinr.ru



Abstract

The present lectures contain an introduction to low energy supersymmetry, a new symmetry that relates bosons and fermions, in particle physics. The Standard Model of fundamental interactions is briefly reviewed, and the motivation to introduce supersymmetry is discussed. The main notions of supersymmetry are introduced. The supersymmetric extension of the Standard Model - the Minimal Supersymmetric Standard Model - is considered in more detail. Phenomenological features of the MSSM as well as possible experimental signatures of SUSY are described. An intriguing situation with the supersymmetric Higgs boson is discussed.

* Lectures given at the European School on High Energy Physics, Aug.-Sept. 2000, Carmulo, Portugal

Contents

1	Introduction. The Standard Model and beyond	3
2	What is supersymmetry? Motivation in particle physics	7
2.1	Unification with gravity	7
2.2	Unification of gauge couplings	8
2.3	Solution of the hierarchy problem	13
2.4	Beyond GUTs: superstring	15
3	Basics of supersymmetry	15
3.1	Algebra of SUSY	16
3.2	Superspace and superfields	17
3.3	Construction of SUSY Lagrangians	20
3.4	The scalar potential	23
3.5	Spontaneous breaking of SUSY	23
4	SUSY generalization of the Standard Model. The MSSM	26
4.1	The field content	27
4.2	Lagrangian of the MSSM	28
4.3	Properties of interactions	29
4.4	Creation and decay of superpartners	31
5	Breaking of SUSY in the MSSM	33
5.1	The hidden sector: four scenarios	33
5.2	The soft terms and the mass formulas	38
5.2.1	Gaugino-higgsino mass terms	38
5.2.2	Squark and slepton masses	38
5.3	The Higgs potential	39
5.4	Renormalization group analysis	41
5.5	Radiative electroweak symmetry breaking	44
6	Constrained MSSM	44
6.1	Parameter space of the MSSM	44
6.2	The choice of constraints	45
6.3	The mass spectrum of superpartners	49
6.4	Experimental signatures at e^+e^- colliders	53
6.5	Experimental signatures at hadron colliders	55
6.6	The lightest superparticle	57
7	The Higgs boson in the SM and the MSSM	58
7.1	Allowed mass range in the SM	58
7.2	SM Higgs production at LEP	61
7.3	The Higgs boson mass in the MSSM	64
7.4	Perspectives of observation	70
8	Conclusion	75
	References	77

1 Introduction. The Standard Model and beyond

The Standard Model (SM) of fundamental interactions describes *strong*, *weak* and *electromagnetic* interactions of elementary particles [1]. It is based on a *gauge principle*, according to which all the forces of Nature are mediated by an exchange of the gauge fields of the corresponding local symmetry group. The symmetry group of the SM is

$$SU_{colour}(3) \otimes SU_{left}(2) \otimes U_{hypercharge}(1), \quad (1.1)$$

whereas the field content is the following:

Gauge sector : Spin = 1

The gauge bosons are spin 1 vector particles belonging to the adjoint representation of the group (1.1). Their quantum numbers with respect to $SU(3) \otimes SU(2) \otimes U(1)$ are

$$\begin{array}{llll} \text{gluons} & G_\mu^a : & (\underline{8}, \underline{1}, 0) & SU_c(3) \quad g_s, \\ \text{intermediate} & W_\mu^i : & (\underline{1}, \underline{3}, 0) & SU_L(2) \quad g, \\ \text{weak bosons} & & & \\ \text{abelian boson} & B_\mu : & (\underline{1}, \underline{1}, 0) & U_Y(1) \quad g', \end{array} \quad (1.2)$$

where the coupling constants are usually denoted by g_s , g and g' , respectively.

Fermion sector : Spin = 1/2

The matter fields are fermions belonging to the fundamental representation of the gauge group. These are believed to be quarks and leptons of at least of three generations. The SM is left-right asymmetric. Left-handed and right-handed fermions have different quantum numbers

$$\begin{array}{ll} \text{quarks} & \\ Q_{\alpha L}^i = \begin{pmatrix} U_\alpha^i \\ D_\alpha^i \end{pmatrix}_L & = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L, \quad \begin{pmatrix} c^i \\ s^i \end{pmatrix}_L, \quad \begin{pmatrix} t^i \\ b^i \end{pmatrix}_L, \quad \dots \quad (\underline{3}, \underline{2}, 1/3) \\ U_{\alpha R}^i & = u_{iR}, \quad c_{iR}, \quad t_{iR}, \quad \dots \quad (\underline{3}^*, \underline{1}, 4/3) \\ D_{\alpha R}^i & = d_{iR}, \quad s_{iR}, \quad b_{iR}, \quad \dots \quad (\underline{3}^*, \underline{1}, -2/3) \\ \text{leptons} & \\ L_{\alpha L} & = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad \dots \quad (\underline{1}, \underline{2}, -1) \\ E_{\alpha R} & = e_R, \quad \mu_R, \quad \tau_R, \quad \dots \quad (\underline{1}, \underline{1}, -2) \end{array} \quad (1.3)$$

$i = 1, 2, 3$ - colour, $\alpha = 1, 2, 3, \dots$ - generation.

Higgs sector : Spin = 0

In the minimal version of the SM there is one doublet of Higgs scalar fields

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \quad (\underline{1}, \underline{2}, -1), \quad (1.4)$$

which is introduced in order to give masses to quarks, leptons and intermediate weak bosons via spontaneous breaking of electroweak symmetry.

In the framework of Quantum Field Theory the SM is described by the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}, \quad (1.5)$$

$$\begin{aligned}
\mathcal{L}_{gauge} = & -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a - \frac{1}{4}W_{\mu\nu}^i W_{\mu\nu}^i - \frac{1}{4}B_{\mu\nu} B_{\mu\nu} \\
& + i\bar{L}_\alpha \gamma^\mu D_\mu L_\alpha + i\bar{Q}_\alpha \gamma^\mu D_\mu Q_\alpha + i\bar{E}_\alpha \gamma^\mu D_\mu E_\alpha \\
& + i\bar{U}_\alpha \gamma^\mu D_\mu U_\alpha + i\bar{D}_\alpha \gamma^\mu D_\mu D_\alpha + (D_\mu H)^\dagger (D_\mu H),
\end{aligned} \tag{1.6}$$

where

$$\begin{aligned}
G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \\
W_{\mu\nu}^i &= \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk} W_\mu^j W_\nu^k, \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
D_\mu L_\alpha &= (\partial_\mu - i\frac{g}{2}\tau^i W_\mu^i + i\frac{g'}{2}B_\mu)L_\alpha, \\
D_\mu E_\alpha &= (\partial_\mu + ig'B_\mu)E_\alpha, \\
D_\mu Q_\alpha &= (\partial_\mu - i\frac{g}{2}\tau^i W_\mu^i - i\frac{g'}{6}B_\mu - i\frac{g_s}{2}\lambda^a G_\mu^a)Q_\alpha, \\
D_\mu U_\alpha &= (\partial_\mu - i\frac{2}{3}g'B_\mu - i\frac{g_s}{2}\lambda^a G_\mu^a)U_\alpha, \\
D_\mu D_\alpha &= (\partial_\mu + i\frac{1}{3}g'B_\mu - i\frac{g_s}{2}\lambda^a G_\mu^a)D_\alpha.
\end{aligned}$$

$$\mathcal{L}_{Yukawa} = y_{\alpha\beta}^L \bar{L}_\alpha E_\beta H + y_{\alpha\beta}^D \bar{Q}_\alpha D_\beta H + y_{\alpha\beta}^U \bar{Q}_\alpha U_\beta \tilde{H} + h.c., \tag{1.7}$$

where $\tilde{H} = i\tau_2 H^\dagger$.

$$\mathcal{L}_{Higgs} = -V = m^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2. \tag{1.8}$$

Here $\{y\}$ are the Yukawa and λ is the Higgs coupling constants, both dimensionless, and m is the only dimensional mass parameter¹.

The Lagrangian of the SM contains the following set of free parameters:

- 3 gauge couplings g_s, g, g' ;
- 3 Yukawa matrices $y_{\alpha\beta}^L, y_{\alpha\beta}^D, y_{\alpha\beta}^U$;
- Higgs coupling constant λ ;
- Higgs mass parameter m^2 ;
- number of matter fields (generations).

All the particles obtain their masses due to spontaneous breaking of $SU_{left}(2)$ symmetry group via a non-zero vacuum expectation value (v.e.v.) of the Higgs field

$$\langle H \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad v = m/\sqrt{\lambda}. \tag{1.9}$$

As a result, the gauge group of the SM is spontaneously broken down to

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \Rightarrow SU_c(3) \otimes U_{EM}(1).$$

¹We use the usual for particle physics units $c = \hbar = 1$

Osaka 2000

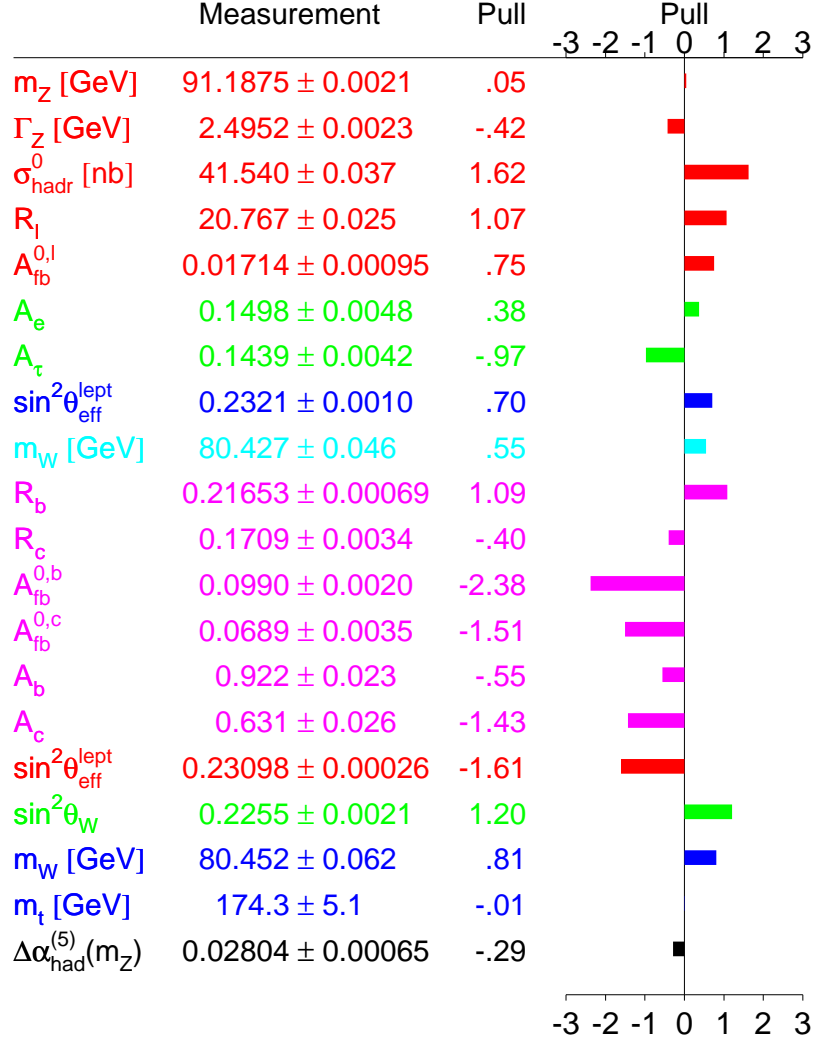


Figure 1: Global Fit of the Standard Model

The physical weak intermediate bosons are linear combinations of the gauge ones

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}, \quad Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3 \quad (1.10)$$

with masses

$$m_W = \frac{1}{\sqrt{2}}gv, \quad m_Z = m_W / \cos \theta_W, \quad \tan \theta_W = g'/g, \quad (1.11)$$

while the photon field

$$\gamma_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3 \quad (1.12)$$

remains massless.

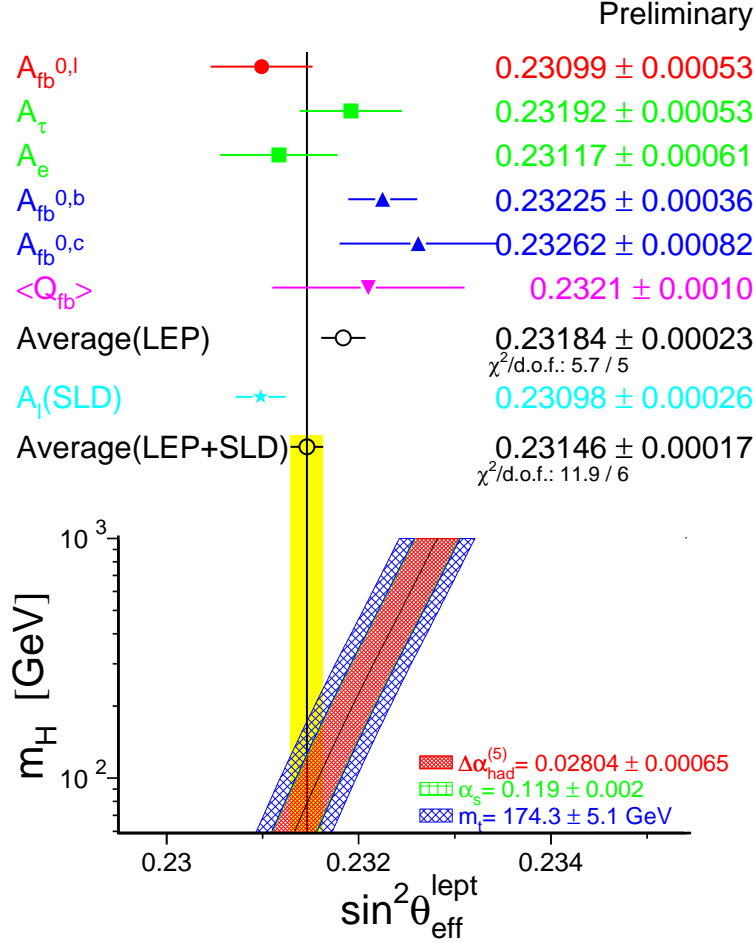


Figure 2: Weak mixing angle and the Higgs boson mass

The matter fields acquire masses proportional to the corresponding Yukawa couplings:

$$M_{\alpha\beta}^u = y_{\alpha\beta}^u v, \quad M_{\alpha\beta}^d = y_{\alpha\beta}^d v, \quad M_{\alpha\beta}^l = y_{\alpha\beta}^l v, \quad m_H = \sqrt{2}m. \quad (1.13)$$

Explicit mass terms in the Lagrangian are forbidden because they are not $SU_{\text{left}}(2)$ symmetric and would destroy the renormalizability of the Standard Model.

The SM has been constructed as a result of numerous efforts both theoretical and experimental. At present, the SM is extraordinary successful, the achieved accuracy of its predictions corresponds to experimental data within 5 % [1, 2]. The combined results of the Global SM fit are shown in Fig.1 [2]. All the particles, except for Higgs boson, have been discovered experimentally. And the mass of the Higgs boson is severely constrained from precision electroweak data (see Fig.2 [2]).

However, the SM has its natural drawbacks and unsolved problems. Among them are

- inconsistency of the SM as a QFT (Landau pole),
- large number of free parameters,
- formal unification of strong and electroweak interactions,

- still unclear mechanism of EW symmetry breaking: The Higgs boson has not yet been observed and it is not clear whether it is fundamental or composite,
- the problem of CP-violation is not well understood including CP-violation in a strong interaction,
- flavour mixing and the number of generations are arbitrary,
- the origin of the mass spectrum is unclear.

The answer to these problems lies beyond the SM. There are two possible ways of going beyond the SM

- \Rightarrow To consider the *same* fundamental fields with *new* interactions. This way leads us to supersymmetry, Grand Unification, String Theory, etc. It seems to be favoured by modern experimental data.
- \Rightarrow To consider *new* fundamental fields with *new* interactions. This way leads us to compositeness, fermion-antifermion condensates, Technicolour, extended Technicolour, preons, etc. It is not favoured by data at the moment.

There are also possible exotic ways out of the SM: gravity at TeV energies, large extra dimensions, brane world, etc. We do not consider them here. In what follows we go along the lines of the first possibility and describe supersymmetry as a nearest option for the new physics on TeV scale.

2 What is supersymmetry? Motivation in particle physics

Supersymmetry or fermion-boson symmetry has not yet been observed in Nature. This is a purely theoretical invention [3]. Its validity in particle physics follows from the common belief in unification. Over 30 years thousands of papers have been written on supersymmetry. For reviews see, e.g. Refs.[4]-[8].

2.1 Unification with gravity

The *general idea* is a unification of all forces of Nature. It defines the *strategy* : increasing unification towards smaller distances up to $l_{Pl} \sim 10^{-33}$ cm including quantum gravity. However, the graviton has spin 2, while the other gauge bosons (photon, gluons, W and Z weak bosons) have spin 1. Therefore, they correspond to different representations of the Poincaré algebra. Attempts to unify all four forces within the same algebra face a problem. Due to no-go theorems [9], unification of spin 2 and spin 1 gauge fields within a unique algebra is forbidden. The only exception from this theorem is supersymmetry algebra. The *uniqueness* of SUSY is due to a strict mathematical statement that algebra of SUSY is the *only* graded (i.e. containing anticommutators as well as commutators) Lie algebra possible within relativistic field theory [9].

If Q is a generator of SUSY algebra, then

$$Q|boson\rangle = |fermion\rangle \quad \text{and} \quad Q|fermion\rangle = |boson\rangle.$$

Hence, starting with the graviton state of spin 2 and acting by SUSY generators we get the following chain of states:

$$spin\ 2 \rightarrow spin\ 3/2 \rightarrow spin\ 1 \rightarrow spin\ 1/2 \rightarrow spin\ 0.$$

Thus, a partial unification of matter (fermions) with forces (bosons) naturally arises from an attempt to unify gravity with other interactions.

SUSY algebra appears as a generalization of Poincaré algebra (see next section) and links together various representations with different spins. The key relation is given by the anticommutator

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma_{\alpha,\dot{\alpha}}^\mu P_\mu.$$

Taking infinitesimal transformations $\delta_\epsilon = \epsilon^\alpha Q_\alpha$, $\bar{\delta}_{\bar{\epsilon}} = \bar{Q}_{\dot{\alpha}} \bar{\epsilon}^{\dot{\alpha}}$, one gets

$$\{\delta_\epsilon, \bar{\delta}_{\bar{\epsilon}}\} = 2(\epsilon\sigma^\mu\bar{\epsilon})P_\mu, \quad (2.1)$$

where ϵ is a transformation parameter. Choosing ϵ to be local, i.e. a function of a space-time point $\epsilon = \epsilon(x)$, one finds from eq.(2.1) that an anticommutator of two SUSY transformations is a local coordinate translation. And a theory which is invariant under the general coordinate transformation is General Relativity. Thus, making SUSY local, one obtains General Relativity, or a theory of gravity, or supergravity [10].

Theoretical attractiveness of SUSY field theories is explained by remarkable properties of SUSY models. This is first of all cancellation of ultraviolet divergencies in rigid SUSY theories which is the origin of

- possible solution of the hierarchy problem in GUTs;
- vanishing of the cosmological constant;
- integrability, allowing for an exact non-perturbative solution.

It is believed that along these lines one can also obtain the unification of all forces of Nature including quantum (super)gravity.

What is essential, the standard concepts of QFT allow SUSY without any further assumptions, it is straightforward to construct the supersymmetric generalization of the SM. Moreover, it can be checked experimentally! In recent years, supersymmetry became a subject of intensive experimental tests. Its predictions can be verified at modern and future colliders.

2.2 Unification of gauge couplings

Since the main motivation for SUSY is related with the unification theory, let us briefly recall the main ideas of the Grand Unification [11].

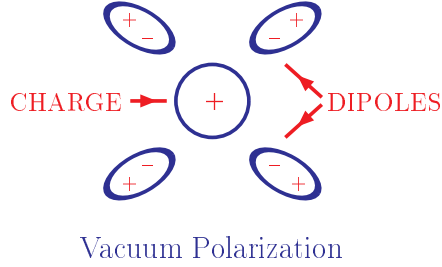
The philosophy of Grand Unification is based on a *hypothesis*: Gauge symmetry increases with energy. Having in mind unification of all forces of Nature on a common basis and neglecting gravity for the time being due to its weakness, the idea of GUTs is the following:

All known interactions are different branches of a unique interaction associated with a simple gauge group. The unification (or splitting) occurs at high energy

Low energy	\Rightarrow	High energy
$SU_c(3) \otimes$ gluons		$SU_L(2) \otimes$ W, Z
quarks		leptons
g_3		g_2
	\Rightarrow	$U_Y(1)$ photon
	\Rightarrow	g_1
	\Rightarrow	G_{GUT} (or G^n + discrete symmetry)
	\Rightarrow	gauge bosons
	\Rightarrow	fermions
	\Rightarrow	g_{GUT}

At first sight this is impossible due to a big difference in the values of the couplings of strong, weak and electromagnetic interactions. However, this is not so. The crucial point here

ELECTRIC SCREENING



MAGNETIC ANTISCREENING

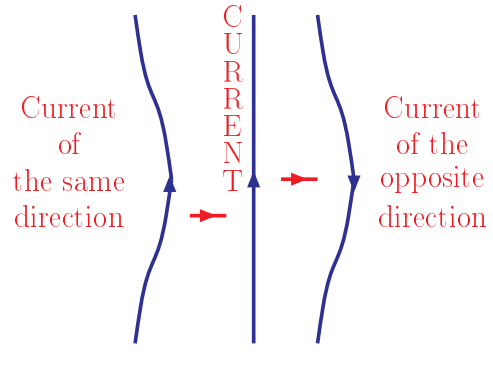


Figure 3: Electric screening and magnetic antiscreening

is the running coupling constants. It is a generic property of quantum field theory which has an analogy in classical physics.

Indeed, consider electric and magnetic phenomena. Let us take some dielectric medium and put a sample electric charge in it. What happens is that the medium is polarized. It contains electric dipoles which are arranged in such a way as to screen the charge (see Fig.3). It is a consequence of the Coulomb law: attraction of the opposite charges and repulsion of the same ones. This is the origin of electric screening.

The opposite situation occurs in a magnetic medium. According to the Biot-Savart law, electric currents of the same direction are attracted to each other, while those of the opposite one are repulsed (see Fig.3). This leads to antiscreening of electric currents in a magnetic medium.

In QFT, the role of the medium is played by the vacuum. Vacuum is polarized due to the presence of virtual pairs of particles in it. The matter fields and transverse quanta of vector fields in this case behave like dipoles in a dielectric medium and cause screening, while the longitudinal quanta of vector fields behave like currents and cause antiscreening. These two effects compete each other (see eq.(2.6) below).

Thus, the couplings become the functions of a distance or an energy scale

$$\alpha_i = \alpha_i\left(\frac{Q^2}{\Lambda^2}\right) = \alpha_i(\text{distance}), \quad \alpha_i \equiv g_i^2/4\pi.$$

This dependence is described by the renormalization group equations and is confirmed experimentally (see Fig.4).

In the SM the strong and weak couplings associated with non-Abelian gauge groups decrease with energy, while the electromagnetic one associated with the Abelian group on the contrary increases. Thus, it becomes possible that at some energy scale they become equal. According to the GUT idea, this equality is not occasional but is a manifestation of a unique origin of these three interactions. As a result of spontaneous symmetry breaking, the unifying group is broken and the unique interaction is splitted into three branches which we call strong, weak and electromagnetic interactions. This happens at a very high energy of an order of $10^{15 \div 16}$ GeV. Of course, this energy is out of the range of accelerators; however, some crucial predictions follow from the very fact of unification.

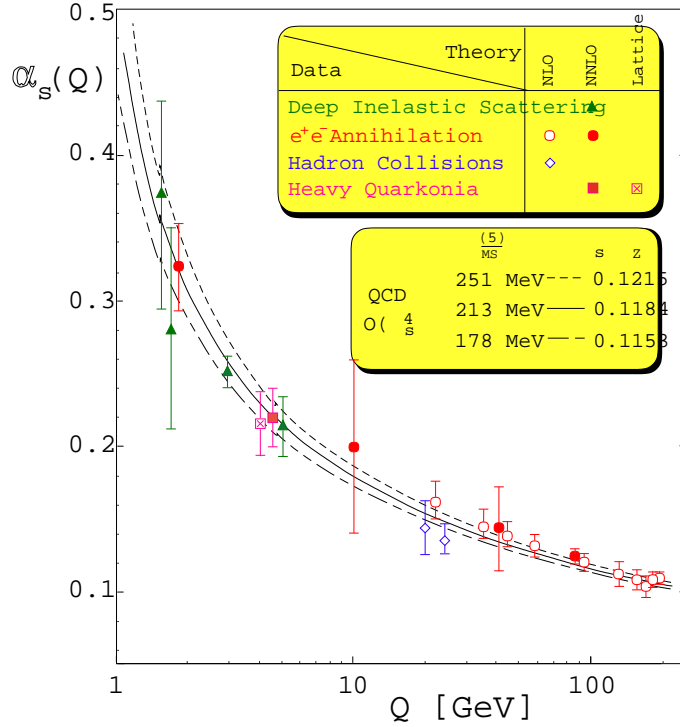


Figure 4: Summary of running of the strong coupling α_s [12]

After the precise measurement of the $SU(3) \times SU(2) \times U(1)$ coupling constants, it has become possible to check the unification numerically.

The three coupling constants to be compared are

$$\begin{aligned}
\alpha_1 &= (5/3)g'^2/(4\pi) = 5\alpha/(3\cos^2\theta_W), \\
\alpha_2 &= g^2/(4\pi) = \alpha/\sin^2\theta_W, \\
\alpha_3 &= g_s^2/(4\pi)
\end{aligned} \tag{2.2}$$

where g' , g and g_s are the usual $U(1)$, $SU(2)$ and $SU(3)$ coupling constants and α is the fine structure constant. The factor of $5/3$ in the definition of α_1 has been included for proper normalization of the generators.

The couplings, when defined as renormalized values including loop corrections require the specification of a renormalization prescription for which the modified minimal subtraction (\overline{MS}) scheme [13] is used.

In this scheme, the world averaged values of the couplings at the Z^0 energy are obtained from a fit to the LEP and Tevatron data [14],[2],[12]:

$$\begin{aligned}
\alpha^{-1}(M_Z) &= 128.978 \pm 0.027 \\
\sin^2\theta_{\overline{MS}} &= 0.23146 \pm 0.00017 \\
\alpha_s &= 0.1184 \pm 0.0031,
\end{aligned} \tag{2.3}$$

that gives

$$\alpha_1(M_Z) = 0.017, \quad \alpha_2(M_Z) = 0.034, \quad \alpha_3(M_Z) = 0.118 \pm 0.003. \tag{2.4}$$

Assuming that the SM is valid up to the unification scale, one can then use the known RG equations for the three couplings. They are the following:

$$\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \frac{\alpha_i}{4\pi}, \quad t = \log\left(\frac{Q^2}{\mu^2}\right), \quad (2.5)$$

where for the SM the coefficients b_i are

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}. \quad (2.6)$$

Here N_{Fam} is the number of generations of matter multiplets and N_{Higgs} is the number of Higgs doublets. We use $N_{Fam} = 3$ and $N_{Higgs} = 1$ for the minimal SM, which gives $b_i = (41/10, -19/6, -7)$.

Notice a positive contribution (screening) from the matter multiplets and negative one (antiscreening) from the gauge fields. For the Abelian group $U(1)$ this contribution is absent due to the absence of a self-interaction of Abelian gauge fields.

The solution to eq.(2.5) is very simple

$$\frac{1}{\tilde{\alpha}_i(Q^2)} = \frac{1}{\tilde{\alpha}_i(\mu^2)} - b_i \log\left(\frac{Q^2}{\mu^2}\right). \quad (2.7)$$

The result is demonstrated in Fig.5 showing the evolution of the inverse of the couplings as a function of the logarithm of energy. In this presentation, the evolution becomes a straight line in first order. The second order corrections are small and do not cause any visible deviation from a straight line. Fig.5 clearly demonstrates that within the SM the coupling constant unification at a single point is impossible. It is excluded by more than 8 standard deviations. This result means that the unification can only be obtained if new physics enters between the electroweak and the Planck scales!

Since we do not know what kind of new physics it may be, there is a lot of arbitrariness. In this situation, some guiding idea is needed. It is very tempting to try to check whether unification is possible within a supersymmetric generalization of the SM. In the SUSY case, the slopes of the RG evolution curves are modified. The coefficients b_i in eq.(2.5) now are

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}, \quad (2.8)$$

where we use $N_{Fam} = 3$ and $N_{Higgs} = 2$ in the minimal SUSY model which gives $b_i = (33/5, 1, -3)$.

It turns out that within the SUSY model a perfect unification can be obtained if the SUSY masses are of an order of 1 TeV. This is shown in Fig.6; the SUSY particles are assumed to effectively contribute to the running of the coupling constants only for energies above the typical SUSY mass scale, which causes the change in the slope of the lines near 1 TeV. From the fit requiring unification one finds for the break point M_{SUSY} and the unification point M_{GUT} [15]

$$\begin{aligned} M_{SUSY} &= 10^{3.4 \pm 0.9 \pm 0.4} \text{ GeV}, \\ M_{GUT} &= 10^{15.8 \pm 0.3 \pm 0.1} \text{ GeV}, \\ \alpha_{GUT}^{-1} &= 26.3 \pm 1.9 \pm 1.0, \end{aligned} \quad (2.9)$$

Unification of the Coupling Constants in the SM and the minimal MSSM

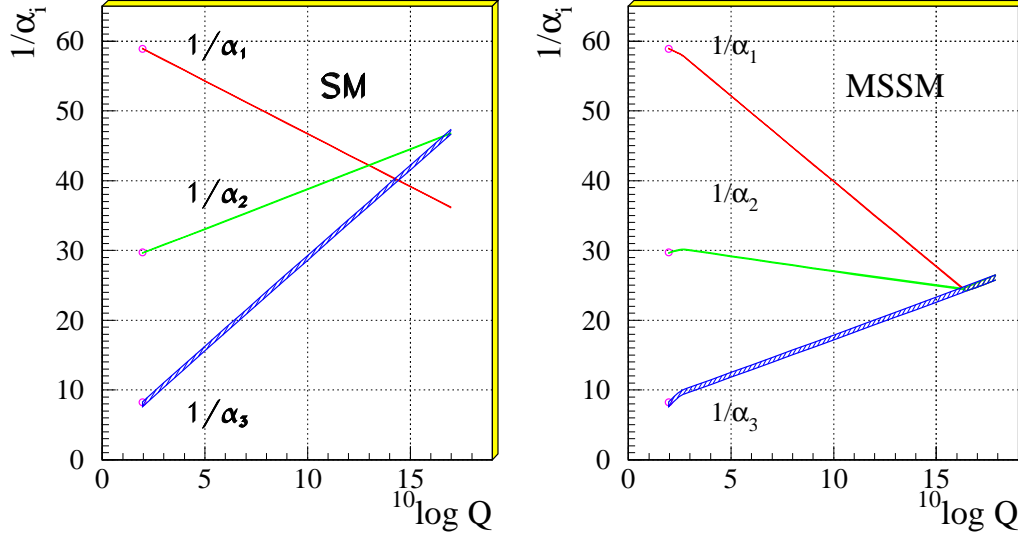


Figure 5: Evolution of the inverse of the three coupling constants in the Standard Model (left) and in the supersymmetric extension of the SM (MSSM) (right). Only in the latter case unification is obtained. The SUSY particles are assumed to contribute only above the effective SUSY scale M_{SUSY} of about 1 TeV, which causes a change in the slope in the evolution of couplings. The thickness of the lines represents the error in the coupling constants [15].

where $\alpha_{GUT} = g_5^2/4\pi$. The first error originates from the uncertainty in the coupling constant, while the second one is due to the uncertainty in the mass splittings between the SUSY particles. The χ^2 distributions of M_{SUSY} and M_{GUT} are shown in Fig.6 [15], where

$$\chi^2 = \sum_{i=1}^3 \frac{(\alpha_i^{-1} - \alpha_{GUT}^{-1})^2}{\sigma_i^2}. \quad (2.10)$$

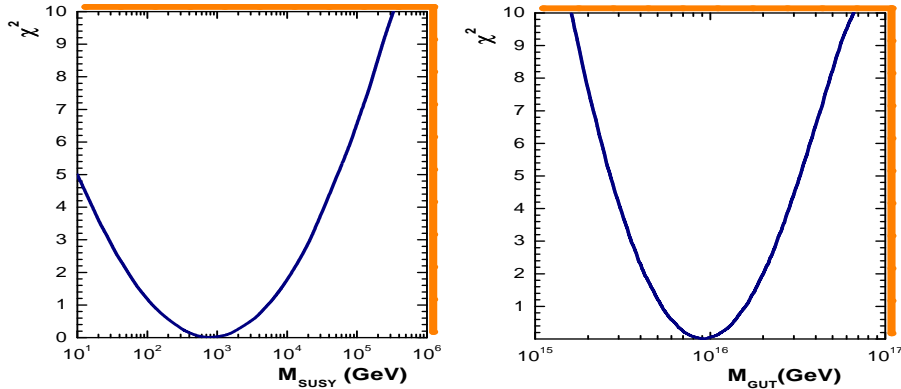


Figure 6: The χ^2 distributions of M_{SUSY} and M_{GUT}

For SUSY models, the dimensional reduction \overline{DR} scheme is a more appropriate renormalization scheme [16]. In this scheme, all thresholds are treated by simple step approximations, and unification occurs if all three α 's meet exactly at one point. This crossing point corresponds to the mass of the heavy gauge bosons. The \overline{MS} and \overline{DR} couplings differ by a small offset

$$\frac{1}{\alpha_i^{DR}} = \frac{1}{\alpha_i^{MS}} - \frac{C_i}{12\pi}, \quad (2.11)$$

where C_i are the quadratic Casimir operators of the group ($C_i = N$ for $SU(N)$ and 0 for $U(1)$ so α_1 remains the same).

This observation was considered as the first "evidence" for supersymmetry, especially since M_{SUSY} was found in the range preferred by the fine-tuning arguments.

It should be noted that the unification of the three curves at a single point is not that trivial as it may seem from the existence of three free parameters (M_{SUSY} , M_{GUT} and α_{GUT}). Out of more than a thousand models tried, only a handful yielded unification. The reason is simple: Introducing new particles one influences all three curves simultaneously, thus giving rise to strong correlations between the slopes of the three lines. For example, adding new generations and/or new Higgs doublets never yields unification! Nevertheless, unification does not prove supersymmetry. The real proof would be the observation of the sparticles.

2.3 Solution of the hierarchy problem

The appearance of two different scales $V \gg v$ in a GUT theory, namely, M_W and M_{GUT} , leads to a very serious problem which is called the *hierarchy problem*. There are two aspects of this problem.

The first one is the very existence of the hierarchy. To get the desired spontaneous symmetry breaking pattern, one needs

$$\frac{m_H}{m_\Sigma} \sim \frac{v}{V} \sim \frac{10^2 \text{ GeV}}{10^{16} \text{ GeV}} \sim 10^{-14} \ll 1, \quad (2.12)$$

where H and Σ are the Higgs fields responsible for the spontaneous breaking of the $SU(2)$ and the GUT groups, respectively.

The question arises of how to get so small number in a natural way. One needs some kind of fine tuning in a theory, and we don't know if there anything behind it.

The second aspect of the hierarchy problem is connected with the preservation of a given hierarchy. Even if we choose the hierarchy like eq.(2.12) the radiative corrections will destroy it! To see this, consider the radiative correction to the light Higgs mass. It is given by the Feynman diagram shown in Fig.7 and is proportional to the mass squared of the heavy particle. This

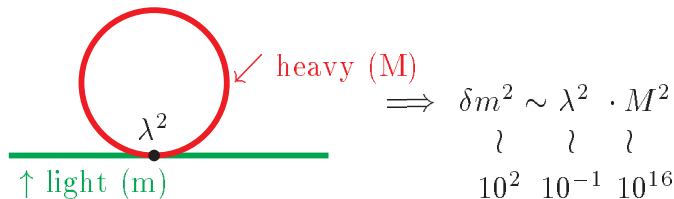


Figure 7: Radiative correction to the light Higgs boson mass

correction obviously spoils the hierarchy if it is not cancelled. This very accurate cancellation with a precision $\sim 10^{-14}$ needs a fine tuning of the coupling constants.

The only known way of achieving this kind of cancellation of quadratic terms (also known as the cancellation of the quadratic divergencies) is supersymmetry. Moreover, SUSY automatically cancels quadratic corrections in all orders of PT. This is due to the contributions of superpartners of ordinary particles. The contribution from boson loops cancels those from the fermion ones because of an additional factor (-1) coming from Fermi statistics, as shown in Fig.8. One can see

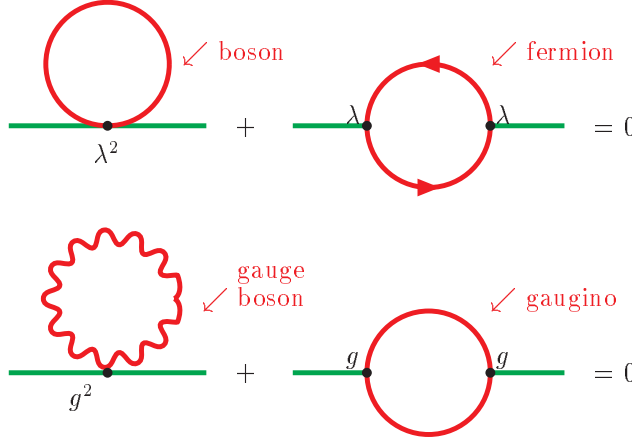


Figure 8: Cancellation of quadratic terms (divergencies)

here two types of contribution. The first line is the contribution of the heavy Higgs boson and its superpartner. The strength of interaction is given by the Yukawa coupling λ . The second line represents the gauge interaction proportional to the gauge coupling constant g with the contribution from the heavy gauge boson and heavy gaugino.

In both the cases the cancellation of quadratic terms takes place. This cancellation is true in the case of unbroken supersymmetry due to the following sum rule relating the masses of superpartners

$$\sum_{bosons} m^2 = \sum_{fermions} m^2 \quad (2.13)$$

and is violated when SUSY is broken. Then, the cancellation is true up to the SUSY breaking scale, M_{SUSY} , since

$$\sum_{bosons} m^2 - \sum_{fermions} m^2 = M_{SUSY}^2, \quad (2.14)$$

which should not be very large (≤ 1 TeV) to make the fine-tuning natural. Indeed, let us take the Higgs boson mass. Requiring for consistency of perturbation theory that the radiative corrections to the Higgs boson mass do not exceed the mass itself gives

$$\delta M_h^2 \sim g^2 M_{SUSY}^2 \sim M_h^2. \quad (2.15)$$

So, if $M_h \sim 10^2$ GeV and $g \sim 10^{-1}$, one needs $M_{SUSY} \sim 10^3$ GeV in order that the relation (2.15) is valid. Thus, we again get the same rough estimate of $M_{SUSY} \sim 1$ TeV as from the gauge coupling unification above. Two requirements match together.

That is why it is usually said that supersymmetry solves the hierarchy problem. Moreover, sometimes it is said that: "There is no GUT without SUSY". However, this is only the second

aspect of the problem, the preservation of the hierarchy. The origin of the hierarchy is the other part of the problem. We show below how SUSY can explain this part as well.

2.4 Beyond GUTs: superstring

Another motivation for supersymmetry follows from even more radical changes of basic ideas related to the ultimate goal of construction of consistent unified theory of everything. At the moment the only viable conception is the superstring theory [17], which pretends to be a self-consistent quantum field theory in a non-perturbative sense allowing exact non-perturbative solutions in the quantum case. In the superstring theory, strings are considered as fundamental objects, closed or open, and are nonlocal in nature. Ordinary particles are considered as string excitation modes. String interactions, which are local, generate proper interactions of usual particles, including gravitational ones.

To be consistent, the string theory should be conformal invariant in D -dimensional target space and have a stable vacuum [18]. The first requirement is valid in classical theory but may be violated by quantum anomalies. Cancellation of quantum anomalies takes place when space-time dimension of a target space equals a critical one. For a bosonic string the critical dimension is $D = 26$, and for a fermionic one it is $D = 10$.

The second requirement is that the massless string excitations (the particles of the SM) are stable. This assumes the absence of tachyons, the states with imaginary mass, which can be guaranteed only in supersymmetric string theories!

Thus, the superstring theory proves to be the only known consistent quantum theory. This serves as justification of research in spite of absence of even a shred of experimental evidence. However, many ingredients of this theory are still unclear.

3 Basics of supersymmetry

Supersymmetry transformations differ from ordinary global transformations as far as they convert bosons into fermions and vice versa. Indeed, if we symbolically write SUSY transformation as

$$\delta B = \varepsilon \cdot f,$$

where B and f are boson and fermion fields, respectively, and ε is an infinitesimal transformation parameter, then from the usual (anti)commutation relations for (fermions) bosons

$$\{f, f\} = 0, \quad [B, B] = 0$$

we immediately find

$$\{\varepsilon, \varepsilon\} = 0.$$

This means that all the generators of SUSY must be *fermionic*, i.e. they must change the spin by a half-odd amount and change the statistics.

3.1 Algebra of SUSY

Combined with the usual Poincaré and internal symmetry algebra the Super-Poincaré Lie algebra contains additional SUSY generators Q_α^i and $\bar{Q}_{\dot{\alpha}}^i$ [3]

$$\begin{aligned}
[P_\mu, P_\nu] &= 0, \\
[P_\mu, M_{\rho\sigma}] &= i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho), \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}), \\
[B_r, B_s] &= iC_{rs}^t B_t, \\
[B_r, P_\mu] &= [B_r, M_{\mu\sigma}] = 0, \\
[Q_\alpha^i, P_\mu] &= [\bar{Q}_{\dot{\alpha}}^i, P_\mu] = 0, \\
[Q_\alpha^i, M_{\mu\nu}] &= \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta^i, \quad [\bar{Q}_{\dot{\alpha}}^i, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}_{\dot{\beta}}^i(\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}}, \\
[Q_\alpha^i, B_r] &= (b_r)_j^i Q_\alpha^j, \quad [\bar{Q}_{\dot{\alpha}}^i, B_r] = -\bar{Q}_{\dot{\alpha}}^j (b_r)_j^i, \\
\{Q_\alpha^i, \bar{Q}_{\dot{\beta}}^j\} &= 2\delta^{ij}(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu, \\
\{Q_\alpha^i, Q_\beta^j\} &= 2\epsilon_{\alpha\beta} Z^{ij}, \quad Z_{ij} = a_{ij}^r b_r, \quad Z^{ij} = Z_{ij}^+, \\
\{\bar{Q}_{\dot{\alpha}}^i, \bar{Q}_{\dot{\beta}}^j\} &= -2\epsilon_{\dot{\alpha}\dot{\beta}} Z^{ij}, \quad [Z_{ij}, \text{anything}] = 0, \\
\alpha, \dot{\alpha} &= 1, 2 \quad i, j = 1, 2, \dots, N.
\end{aligned} \tag{3.1}$$

Here P_μ and $M_{\mu\nu}$ are four-momentum and angular momentum operators, respectively, B_r are the internal symmetry generators, Q^i and \bar{Q}^i are the spinorial SUSY generators and Z_{ij} are the so-called central charges; $\alpha, \dot{\alpha}, \beta, \dot{\beta}$ are the spinorial indices. In the simplest case one has one spinor generator Q_α (and the conjugated one $\bar{Q}_{\dot{\alpha}}$) that corresponds to an ordinary or N=1 supersymmetry. When $N > 1$ one has an extended supersymmetry.

A natural question arises: how many SUSY generators are possible, i.e. what is the value of N ? To answer this question, consider massless states [5]. Let us start with the ground state labeled by energy and helicity, i.e. projection of a spin on the direction of momenta, and let it be annihilated by Q_i

$$\text{Vacuum} = |E, \lambda \rangle, \quad Q_i |E, \lambda \rangle = 0.$$

Then one and more particle states can be constructed with the help of a creation operators as

State	Expression	# of States
vacuum	$ E, \lambda \rangle$	1
1 – particle state	$\bar{Q}_i E, \lambda \rangle = E, \lambda + 1/2 \rangle_i$	$\binom{N}{1} = N$
2 – particle state	$\bar{Q}_i \bar{Q}_j E, \lambda \rangle = E, \lambda + 1 \rangle_{ij}$	$\binom{N}{2} = \frac{N(N-1)}{2}$
...
N – particle state	$\bar{Q}_1 \bar{Q}_2 \dots \bar{Q}_N E, \lambda \rangle = E, \lambda + N/2 \rangle$	$\binom{N}{N} = 1$
Total # of States	$\sum_{k=0}^N \binom{N}{k} = 2^N = 2^{N-1} \text{ bosons} + 2^{N-1} \text{ fermions},$	

where the energy E is not changed, since according to (3.1) the operators \bar{Q}_i commute with the Hamiltonian.

Thus, one has a sequence of bosonic and fermionic states and the total number of bosons equals that of fermions. This is a generic property of any supersymmetric theory. However,

in CPT invariant theories the number of states is doubled, since CPT transformation changes the sign of helicity. Hence, in CPT invariant theories, one has to add the states with opposite helicity to the above mentioned ones.

Consider some examples. Let us take $N = 1$ and $\lambda = 0$. Then one has the following set of states:

$$\begin{array}{ccc}
 \text{helicity} & 0 & 1/2 \\
 N = 1 \quad \lambda = 0 & & \\
 \# \text{ of states} & 1 & 1
 \end{array}
 \xrightarrow{CPT}
 \begin{array}{ccc}
 \text{helicity} & 0 & -1/2 \\
 & & \\
 \# \text{ of states} & 1 & 1
 \end{array}$$

Hence, a complete $N = 1$ multiplet is

$$\begin{array}{ccc}
 N = 1 & \text{helicity} & -1/2 \quad 0 \quad 1/2 \\
 & \# \text{ of states} & 1 \quad 2 \quad 1
 \end{array}$$

which contains one complex scalar and one spinor with two helicity states.

This is an example of the so-called self-conjugated multiplet. There are also self-conjugated multiplets with $N > 1$ corresponding to extended supersymmetry. Two particular examples are the $N = 4$ super Yang-Mills multiplet and the $N = 8$ supergravity multiplet

$$\begin{array}{ccc}
 N = 4 & \text{SUSY YM} & \text{helicity} & -1 & -1/2 & 0 & 1/2 & 1 \\
 & \lambda = -1 & \# \text{ of states} & 1 & 4 & 6 & 4 & 1
 \end{array}$$

$$\begin{array}{ccc}
 N = 8 & \text{SUGRA} & \text{helicity} & -2 & -3/2 & -1 & -1/2 & 0 & 1/2 & 1 & 3/2 & 2 \\
 & \lambda = -2 & \# \text{ of states} & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1
 \end{array}$$

One can see that the multiplets of extended supersymmetry are very rich and contain a vast number of particles.

The constraint on the number of SUSY generators comes from a requirement of consistency of the corresponding QFT. The number of supersymmetries and the maximal spin of the particle in the multiplet are related by

$$N \leq 4S,$$

where S is the maximal spin. Since the theories with spin greater than 1 are non-renormalizable and the theories with spin greater than 5/2 have no consistent coupling to gravity, this imposes a constraint on the number of SUSY generators

$$\begin{array}{ll}
 N \leq 4 & \text{for renormalizable theories (YM),} \\
 N \leq 8 & \text{for (super)gravity.}
 \end{array}$$

In what follows, we shall consider simple supersymmetry, or $N = 1$ supersymmetry, contrary to extended supersymmetries with $N > 1$. In this case, one has two types of supermultiplets: the so-called chiral multiplet with $\lambda = 0$, which contains two physical states (ϕ, ψ) with spin 0 and 1/2, respectively, and the vector multiplet with $\lambda = 1/2$, which also contains two physical states (λ, A_μ) with spin 1/2 and 1, respectively.

3.2 Superspace and superfields

An elegant formulation of supersymmetry transformations and invariants can be achieved in the framework of superspace [7]. Superspace differs from the ordinary Euclidean (Minkowski) space

by adding of two new coordinates, θ_α and $\bar{\theta}_{\dot{\alpha}}$, which are Grassmannian, i.e. anticommuting, variables

$$\{\theta_\alpha, \theta_\beta\} = 0, \quad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0, \quad \theta_\alpha^2 = 0, \quad \bar{\theta}_{\dot{\alpha}}^2 = 0, \quad \alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2.$$

Thus, we go from space to superspace

$$\begin{array}{ccc} \text{Space} & \Rightarrow & \text{Superspace} \\ x_\mu & & x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}} \end{array}$$

A SUSY group element can be constructed in superspace in the same way as an ordinary translation in the usual space

$$G(x, \theta, \bar{\theta}) = e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta} \bar{Q})}. \quad (3.2)$$

It leads to a supertranslation in superspace

$$\begin{array}{ccc} x_\mu & \rightarrow & x_\mu + i\theta\sigma_\mu\bar{\varepsilon} - i\varepsilon\sigma_\mu\bar{\theta}, \\ \theta & \rightarrow & \theta + \varepsilon, \\ \bar{\theta} & \rightarrow & \bar{\theta} + \bar{\varepsilon}, \end{array} \quad (3.3)$$

where ε and $\bar{\varepsilon}$ are Grassmannian transformation parameters. From eq.(3.3) one can easily obtain the representation for the supercharges (3.1) acting on the superspace

$$Q_\alpha = \frac{\partial}{\partial\theta_\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i\theta_\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu. \quad (3.4)$$

Taking the Grassmannian transformation parameters to be local, or space-time dependent, one gets a local translation. As has already been mentioned, this leads to a theory of (super) gravity.

To define the fields on a superspace, consider representations of the Super-Poincaré group (3.1) [5]. The simplest one is a scalar superfield $F(x, \theta, \bar{\theta})$ which is SUSY invariant. Its Taylor expansion in θ and $\bar{\theta}$ has only several terms due to the nilpotent character of Grassmannian parameters. However, this superfield is a reducible representation of SUSY. To get an irreducible one, we define a *chiral* superfield which obeys the equation

$$\bar{D}F = 0, \quad \text{where } \bar{D} = -\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma^\mu\partial_\mu \quad (3.5)$$

is a superspace covariant derivative.

For the chiral superfield Grassmannian Taylor expansion looks like ($y = x + i\theta\sigma\bar{\theta}$)

$$\begin{aligned} \Phi(y, \theta) &= A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ &= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square A(x) \\ &+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x). \end{aligned} \quad (3.6)$$

The coefficients are ordinary functions of x being the usual fields. They are called the *components* of a superfield. In eq.(3.6) one has 2 bosonic (complex scalar field A) and 2 fermionic (Weyl spinor field ψ) degrees of freedom. The component fields A and ψ are called the *superpartners*. The field F is an *auxiliary* field, it has the “wrong” dimension and has no physical meaning. It is needed to close the algebra (3.1). One can get rid of the auxiliary fields with the help of equations of motion.

Thus, a superfield contains an equal number of bosonic and fermionic degrees of freedom. Under SUSY transformation they convert into one another

$$\begin{aligned}\delta_\varepsilon A &= \sqrt{2}\varepsilon\psi, \\ \delta_\varepsilon\psi &= i\sqrt{2}\sigma^\mu\bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F, \\ \delta_\varepsilon F &= i\sqrt{2}\bar{\varepsilon}\sigma^\mu\partial_\mu\psi.\end{aligned}\tag{3.7}$$

Notice that the variation of the F -component is a total derivative, i.e. it vanishes when integrated over the space-time.

One can also construct an antichiral superfield Φ^+ obeying the equation

$$D\Phi^+ = 0, \quad \text{with } D = \frac{\partial}{\partial\theta} + i\sigma^\mu\bar{\theta}\partial_\mu.$$

The product of chiral (antichiral) superfields Φ^2, Φ^3 , etc is also a chiral (antichiral) superfield, while the product of chiral and antichiral ones $\Phi^+\Phi$ is a general superfield.

For any arbitrary function of chiral superfields one has

$$\begin{aligned}\mathcal{W}(\Phi_i) &= \mathcal{W}(A_i + \sqrt{2}\theta\psi_i + \theta\theta F) \\ &= \mathcal{W}(A_i) + \frac{\partial\mathcal{W}}{\partial A_i}\sqrt{2}\theta\psi_i + \theta\theta\left(\frac{\partial\mathcal{W}}{\partial A_i}F_i - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i\partial A_j}\psi_i\psi_j\right).\end{aligned}\tag{3.8}$$

The \mathcal{W} is usually referred to as a superpotential which replaces the usual potential for the scalar fields.

To construct the gauge invariant interactions, one needs a real vector superfield $V = V^+$. It is not chiral but rather a general superfield with the following Grassmannian expansion:

$$\begin{aligned}V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) \\ &+ \frac{i}{2}\theta\theta[M(x) + iN(x)] - \frac{i}{2}\bar{\theta}\bar{\theta}[M(x) - iN(x)] \\ &- \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}[\lambda(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)] \\ &- i\bar{\theta}\bar{\theta}\theta[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) + \frac{1}{2}\square C(x)].\end{aligned}\tag{3.9}$$

The physical degrees of freedom corresponding to a real vector superfield V are the vector gauge field v_μ and the Majorana spinor field λ . All other components are unphysical and can be eliminated. Indeed, under the Abelian (super)gauge transformation the superfield V is transformed as

$$V \rightarrow V + \Phi + \Phi^+,$$

where Φ and Φ^+ are some chiral superfields. In components it looks like

$$\begin{aligned}C &\rightarrow C + A + A^*, \\ \chi &\rightarrow \chi - i\sqrt{2}\psi, \\ M + iN &\rightarrow M + iN - 2iF, \\ v_\mu &\rightarrow v_\mu - i\partial_\mu(A - A^*), \\ \lambda &\rightarrow \lambda, \\ D &\rightarrow D,\end{aligned}\tag{3.10}$$

and corresponds to ordinary gauge transformations for physical components. According to eq.(3.10), one can choose a gauge (the Wess-Zumino gauge [19]) where $C = \chi = M = N = 0$, leaving one with only physical degrees of freedom except for the auxiliary field D . In this gauge

$$\begin{aligned} V &= -\theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \\ V^2 &= -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}v_\mu(x)v^\mu(x), \\ V^3 &= 0, \quad \text{etc.} \end{aligned} \quad (3.11)$$

One can define also a field strength tensor (as analog of $F_{\mu\nu}$ in gauge theories)

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar{D}^2 e^V D_\alpha e^{-V}, \\ \bar{W}_{\dot{\alpha}} &= -\frac{1}{4}D^2 e^V \bar{D}_{\dot{\alpha}} e^{-V}, \end{aligned} \quad (3.12)$$

which is a polynomial in the Wess-Zumino gauge. (Here D s are the supercovariant derivatives.)

The strength tensor is a chiral superfield

$$\bar{D}_{\dot{\beta}}W_\alpha = 0, \quad D_\beta\bar{W}_{\dot{\alpha}} = 0.$$

In the Wess-Zumino gauge it is a polynomial over component fields:

$$W_\alpha = T^a \left(-i\lambda_\alpha^a + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu}^a + \theta^2\sigma^\mu D_\mu\bar{\lambda}^a \right), \quad (3.13)$$

where

$$F_{\mu\nu}^a = \partial_\mu v_\nu^a - \partial_\nu v_\mu^a + f^{abc}v_\mu^b v_\nu^c, \quad D_\mu\bar{\lambda}^a = \partial\bar{\lambda}^a + f^{abc}v_\mu^b\bar{\lambda}^c.$$

In Abelian case eqs.(3.12) are simplified and take form

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2 \bar{D}_{\dot{\alpha}} V.$$

3.3 Construction of SUSY Lagrangians

Let us start with the Lagrangian which has no local gauge invariance. In the superfield notation SUSY invariant Lagrangians are the polynomials of superfields. Having in mind that for component fields one should have ordinary terms and the above mentioned property of SUSY invariance of the highest dimension components of a superfield, the general SUSY invariant Lagrangian has the form

$$\mathcal{L} = \Phi_i^+ \Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} + [(\lambda_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k)|_{\theta\theta} + h.c.]. \quad (3.14)$$

Hereafter the vertical line means the corresponding term of a Taylor expansion.

The first term is a kinetic term. It contains both the chiral and antichiral superfields Φ_i and Φ_i^+ , respectively, and is a function of Grassmannian parameters θ and $\bar{\theta}$. Being expanded over θ and $\bar{\theta}$ it leads to the usual kinetic terms for the corresponding component fields.

The terms in the bracket form the superpotential. It is composed of the chiral fields only (plus the hermitian conjugated counterpart composed of antichiral superfields) and is a chiral superfield. Since the products of a chiral superfield and antichiral one produce a general superfield, they are not allowed in a superpotential. The last coefficient of its expansion over the

parameter θ is supersymmetrically invariant and gives the usual potential after getting rid of the auxiliary fields, as it will be clear later.

The Lagrangian (3.14) can be written in a much more elegant way in superspace. The same way as an ordinary action is an integral over space-time of Lagrangian density, in supersymmetric case the action is an integral over the superspace. The space-time Lagrangian density then is [5, 6, 7]

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \Phi_i^+ \Phi_i + \int d^2\theta [\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k] + h.c. \quad (3.15)$$

where the first part is a kinetic term and the second one is a superpotential \mathcal{W} . Here instead of taking the proper components we use integration over the superspace according to the rules of Grassmannian integration [20]

$$\int d\theta_\alpha = 0, \quad \int \theta_\alpha d\theta_\beta = \delta_{\alpha\beta}.$$

Performing explicit integration over the Grassmannian parameters, we get from eq.(3.15)

$$\begin{aligned} \mathcal{L} = & i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \square A_i + F_i^* F_i \\ & + [\lambda_i F_i + m_{ij} (A_i F_j - \frac{1}{2} \psi_i \psi_j) + y_{ijk} (A_i A_j F_k - \psi_i \psi_j A_k) + h.c.]. \end{aligned} \quad (3.16)$$

The last two terms are the interaction ones. To obtain a familiar form of the Lagrangian, we have to solve the constraints

$$\frac{\partial \mathcal{L}}{\partial F_k^*} = F_k + \lambda_k^* + m_{ik}^* A_i^* + y_{ijk}^* A_i^* A_j^* = 0, \quad (3.17)$$

$$\frac{\partial \mathcal{L}}{\partial F_k} = F_k^* + \lambda_k + m_{ik} A_i + y_{ijk} A_i A_j = 0. \quad (3.18)$$

Expressing the auxiliary fields F and F^* from these equations, one finally gets

$$\begin{aligned} \mathcal{L} = & i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \square A_i - \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j \\ & - y_{ijk} \psi_i \psi_j A_k - y_{ijk}^* \bar{\psi}_i \bar{\psi}_j A_k^* - V(A_i, A_j), \end{aligned} \quad (3.19)$$

where the scalar potential $V = F_k^* F_k$. We will return to the discussion of the form of the scalar potential in SUSY theories later.

Consider now the gauge invariant SUSY Lagrangians. They should contain gauge invariant interaction of the matter fields with the gauge ones and the kinetic term and the self-interaction of the gauge fields.

Let us start with the gauge field kinetic terms. In the Wess-Zumino gauge one has

$$W^\alpha W_\alpha|_{\theta\theta} = -2i\lambda\sigma^\mu D_\mu \bar{\lambda} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D^2 + i\frac{1}{4} F^{\mu\nu} F^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}, \quad (3.20)$$

where $D_\mu = \partial_\mu + ig[v_\mu, \cdot]$ is the usual covariant derivative and the last, the so-called topological θ term,² is the total derivative.

The gauge invariant Lagrangian now has a familiar form

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} \int d^2\theta W^\alpha W_\alpha + \frac{1}{4} \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \\ = & \frac{1}{2} D^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\lambda\sigma^\mu D_\mu \bar{\lambda}. \end{aligned} \quad (3.21)$$

² Terminology comes from the θ term of QCD [21] and has nothing to do with the Grassmannian parameter θ .

To obtain a gauge-invariant interaction with matter chiral superfields, consider their gauge transformation (Abelian)

$$\Phi \rightarrow e^{-ig\Lambda}\Phi, \quad \Phi^+ \rightarrow \Phi^+ e^{ig\Lambda^+}, \quad V \rightarrow V + i(\Lambda - \Lambda^+),$$

where Λ is a gauge parameter (chiral superfield).

It is clear now how to construct both the SUSY and gauge invariant kinetic term (compare with the covariant derivative in a usual gauge theory)

$$\Phi_i^+ \Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} \Rightarrow \Phi_i^+ e^{gV} \Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} \quad (3.22)$$

A complete SUSY and gauge invariant Lagrangian then looks like

$$\begin{aligned} \mathcal{L}_{inv} &= \frac{1}{4} \int d^2\theta W^\alpha W_\alpha + \frac{1}{4} \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} + \int d^2\theta d^2\bar{\theta} \Phi_i^+ e^{gV} \Phi_i \\ &+ \int d^2\theta \left(\frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k \right) + h.c. \end{aligned} \quad (3.23)$$

In particular, the SUSY generalization of QED looks as follows:

$$\begin{aligned} \mathcal{L}_{SUSY \text{ QED}} &= \frac{1}{4} \int d^2\theta W^\alpha W_\alpha + \frac{1}{4} \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \\ &+ \int d^4\theta (\Phi_+^+ e^{gV} \Phi_+ + \Phi_-^+ e^{-gV} \Phi_-) \\ &+ \int d^2\theta m \Phi_+ \Phi_- + \int d^2\bar{\theta} m \Phi_+^+ \Phi_-^+, \end{aligned} \quad (3.24)$$

where two superfields Φ_+ and Φ_- have been introduced in order to have both left- and right-handed fermions.

The non-Abelian generalization is straightforward

$$\begin{aligned} \mathcal{L}_{SUSY \text{ YM}} &= \frac{1}{4} \int d^2\theta \text{Tr}(W^\alpha W_\alpha) + \frac{1}{4} \int d^2\bar{\theta} \text{Tr}(\bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}) \\ &+ \int d^2\theta d^2\bar{\theta} \bar{\Phi}_{ia} (e^{gV})_b^a \Phi_i^b + \int d^2\theta \mathcal{W}(\Phi_i) + \int d^2\bar{\theta} \bar{\mathcal{W}}(\bar{\Phi}_i), \end{aligned} \quad (3.25)$$

where \mathcal{W} is a superpotential, which should be invariant under the group of symmetry of a particular model.

In terms of component fields the above Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_{SUSY \text{ YM}} &= -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - i\lambda^a \sigma^\mu D_\mu \bar{\lambda}^a + \frac{1}{2} D^a D^a \\ &+ (\partial_\mu A_i - igv_\mu^a T^a A_i)^\dagger (\partial_\mu A_i - igv_\mu^a T^a A_i) - i\bar{\psi}_i \bar{\sigma}^\mu (\partial_\mu \psi_i - igv_\mu^a T^a \psi_i) \\ &- D^a A_i^\dagger T^a A_i - i\sqrt{2} A_i^\dagger T^a \lambda^a \psi_i + i\sqrt{2} \bar{\psi}_i T^a A_i \bar{\lambda}^a + F_i^\dagger F_i \\ &+ \frac{\partial \mathcal{W}}{\partial A_i} F_i + \frac{\partial \bar{\mathcal{W}}}{\partial A_i^\dagger} F_i^\dagger - \frac{1}{2} \frac{\partial^2 \mathcal{W}}{\partial A_i \partial A_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 \bar{\mathcal{W}}}{\partial A_i^\dagger \partial A_j^\dagger} \bar{\psi}_i \bar{\psi}_j. \end{aligned} \quad (3.26)$$

Integrating out the auxiliary fields D^a and F_i , one reproduces the usual Lagrangian.

3.4 The scalar potential

Contrary to the SM, where the scalar potential is arbitrary and is defined only by the requirement of the gauge invariance, in supersymmetric theories it is completely defined by the superpotential. It consists of the contributions from the D -terms and F -terms. The kinetic energy of the gauge fields (recall eq.(3.21) yields the $1/2 D^a D^a$ term, and the matter-gauge interaction (recall eq.(3.23) yields the $g D^a T_{ij}^a A_i^* A_j$ one. Together they give

$$\mathcal{L}_D = \frac{1}{2} D^a D^a + g D^a T_{ij}^a A_i^* A_j. \quad (3.27)$$

The equation of motion reads

$$D^a = -g T_{ij}^a A_i^* A_j. \quad (3.28)$$

Substituting it back into eq.(3.27) yields the D -term part of the potential

$$\mathcal{L}_D = -\frac{1}{2} D^a D^a \quad \implies V_D = \frac{1}{2} D^a D^a, \quad (3.29)$$

where D is given by eq.(3.28).

The F -term contribution can be derived from the matter field self-interaction eq.(3.16). For a general type superpotential W one has

$$\mathcal{L}_F = F_i^* F_i + \left(\frac{\partial W}{\partial A_i} F_i + h.c. \right). \quad (3.30)$$

Using the equations of motion for the auxiliary field F_i

$$F_i^* = -\frac{\partial W}{\partial A_i} \quad (3.31)$$

yields

$$\mathcal{L}_F = -F_i^* F_i \quad \implies V_F = F_i^* F_i, \quad (3.32)$$

where F is given by eq.(3.31). The full potential is the sum of the two contributions

$$V = V_D + V_F. \quad (3.33)$$

Thus, the form of the Lagrangian is practically fixed by symmetry requirements. The only freedom is the field content, the value of the gauge coupling g , Yukawa couplings y_{ijk} and the masses. Because of the renormalizability constraint $V \leq A^4$ the superpotential should be limited by $\mathcal{W} \leq \Phi^3$ as in eq.(3.15). All members of a supermultiplet have the same masses, i.e. bosons and fermions are degenerate in masses. This property of SUSY theories contradicts the phenomenology and requires supersymmetry breaking.

3.5 Spontaneous breaking of SUSY

Since supersymmetric algebra leads to mass degeneracy in a supermultiplet, it should be broken to explain the absence of superpartners at modern energies. There are several ways of supersymmetry breaking. It can be broken either explicitly or spontaneously. Performing SUSY breaking one has to be careful not to spoil the cancellation of quadratic divergencies which allows one to solve the hierarchy problem. This is achieved by spontaneous breaking of SUSY.

Apart from non-supersymmetric theories in SUSY models the energy is always nonnegative definite. Indeed, according to quantum mechanics

$$E = \langle 0 | H | 0 \rangle$$

and due to SUSY algebra eq.(3.1)

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu,$$

taking into account that $\text{tr}(\sigma^\mu P_\mu) = 2P_0$, one gets

$$E = \frac{1}{4} \sum_{\alpha=1,2} \langle 0 | \{Q_\alpha, \bar{Q}_\alpha\} | 0 \rangle = \frac{1}{4} \sum_{\alpha} |Q_\alpha | 0 \rangle|^2 \geq 0.$$

Hence

$$E = \langle 0 | H | 0 \rangle \neq 0 \quad \text{if and only if} \quad Q_\alpha | 0 \rangle \neq 0.$$

Therefore, supersymmetry is spontaneously broken, i.e. vacuum is not invariant ($Q_\alpha | 0 \rangle \neq 0$), *if and only if* the minimum of the potential is positive (*i.e.* $E > 0$).

The situation is illustrated in Fig.9. The SUSY ground state has $E = 0$, while a non-SUSY one has $E > 0$. On the right-hand side a non-SUSY potential is shown. It does not appear even in spontaneously broken SUSY theories. However, just this type of the potential is used for spontaneous breaking of the gauge invariance via the Higgs mechanism. This property has crucial consequences for the spontaneous breaking of the gauge invariance. Indeed, as will be seen later, in the MSSM spontaneous breaking of $SU(2)$ invariance takes place only after SUSY is broken.

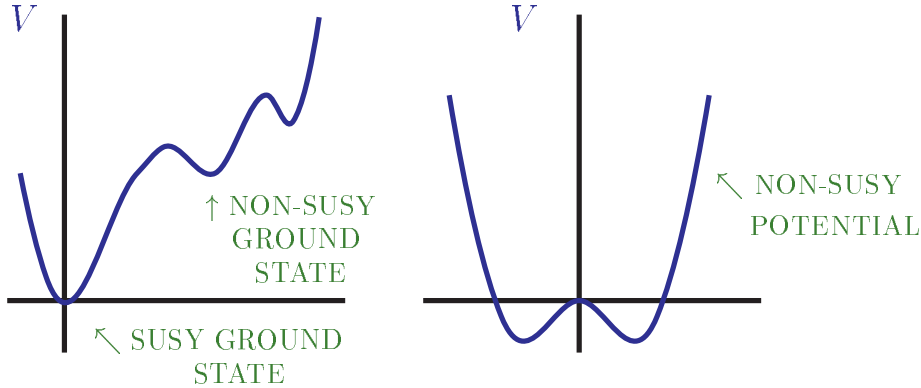


Figure 9: Scalar potential in supersymmetric and non-supersymmetric theories

Spontaneous breaking of supersymmetry is achieved in the same way as the electroweak symmetry breaking. One introduces the field whose vacuum expectation value is nonzero and breaks the symmetry. However, due to a special character of SUSY, this should be a superfield whose auxiliary F and D components acquire nonzero v.e.v.'s. Thus, among possible spontaneous SUSY breaking mechanisms one distinguishes the F and D ones.

i) Fayet-Iliopoulos (D -term) mechanism [22].

In this case the, the linear D -term is added to the Lagrangian

$$\Delta\mathcal{L} = \xi V|_{\theta\theta\bar{\theta}\bar{\theta}} = \xi \int d^4\theta V. \quad (3.34)$$

It is gauge and SUSY invariant by itself; however, it may lead to spontaneous breaking of both of them depending on the value of ξ . We show in Fig.10a the sample spectrum for two chiral matter multiplets. The drawback of this mechanism is the necessity of $U(1)$ gauge invariance.

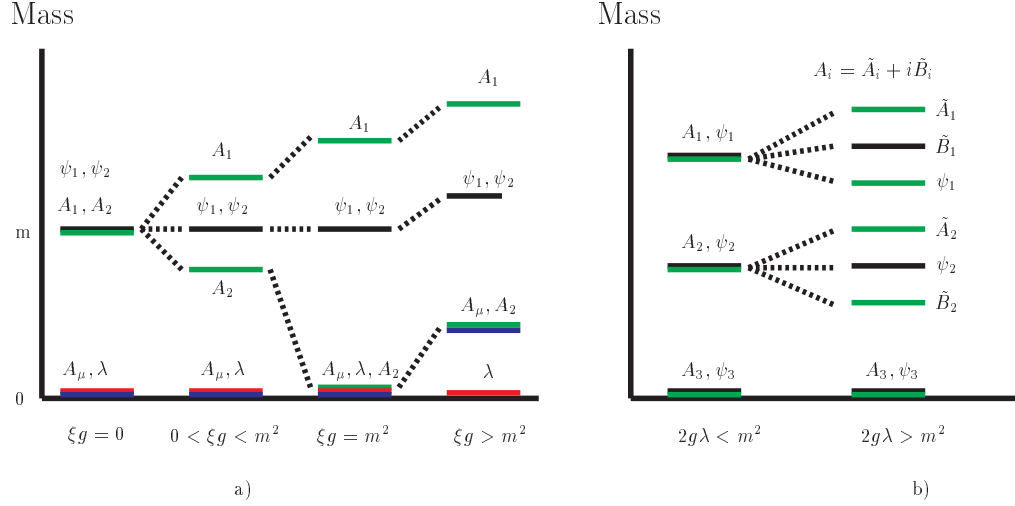


Figure 10: Spectrum of spontaneously broken SUSY theories

It can be used in SUSY generalizations of the SM but not in GUTs.

The mass spectrum also causes some troubles since the following sum rule is always valid

$$\sum_{boson\ states} m_i^2 = \sum_{fermion\ states} m_i^2, \quad (3.35)$$

which is bad for phenomenology.

ii) O'RaiFeartaigh (F -term) mechanism [23].

In this case, several chiral fields are needed and the superpotential should be chosen in a way that trivial zero v.e.v.s for the auxiliary F -fields be absent. For instance, choosing the superpotential to be

$$\mathcal{W}(\Phi) = \lambda\Phi_3 + m\Phi_1\Phi_2 + g\Phi_3\Phi_1^2,$$

one gets the equations for the auxiliary fields

$$\begin{aligned} F_1^* &= mA_2 + 2gA_1A_3, \\ F_2^* &= mA_1, \\ F_3^* &= \lambda + gA_1^2, \end{aligned}$$

which have no solutions with $\langle F_i \rangle = 0$ and SUSY is spontaneously broken. The sample spectrum is shown in Fig.10b.

The drawbacks of this mechanism is a lot of arbitrariness in the choice of potential. The sum rule (3.35) is also valid here.

Unfortunately, none of these mechanisms explicitly works in SUSY generalizations of the SM. None of the fields of the SM can develop nonzero v.e.v.s for their F or D components without breaking $SU(3)$ or $U(1)$ gauge invariance since they are not singlets with respect to these groups. This requires the presence of extra sources of spontaneous SUSY breaking, which we consider below. They are based, however, on the same F and D mechanisms.

4 SUSY generalization of the Standard Model. The MSSM

As has been already mentioned, in SUSY theories the number of bosonic degrees of freedom equals that of fermionic. At the same time, in the SM one has 28 bosonic and 90 fermionic degrees of freedom (with massless neutrino, otherwise 96). So the SM is to a great extent non-supersymmetric. Trying to add some new particles to supersymmetrize the SM, one should take into account the following observations:

1. There are no fermions with quantum numbers of the gauge bosons;
2. Higgs fields have nonzero v.e.v.s; hence they cannot be superpartners of quarks and leptons since this would induce spontaneous violation of baryon and lepton numbers;
3. One needs at least two complex chiral Higgs multiplets to give masses to Up and Down quarks.

The latter is due to the form of a superpotential and chirality of matter superfields. Indeed, the superpotential should be invariant under the $SU(3) \times SU(2) \times U(1)$ gauge group. If one looks at the Yukawa interaction in the Standard Model, eq.(1.7), one finds that it is indeed $U(1)$ invariant since the sum of hypercharges in each vertex equals zero. In the last term this is achieved by taking the conjugated Higgs doublet $\tilde{H} = i\tau_2 H^\dagger$ instead of H . However, in SUSY H is a chiral superfield and hence a superpotential, which is constructed out of chiral fields, can contain only H but not \tilde{H} which is an antichiral superfield.

Another reason for the second Higgs doublet is related to chiral anomalies. It is known that chiral anomalies spoil the gauge invariance and, hence, the renormalizability of the theory. They are canceled in the SM between quarks and leptons in each generation.

Indeed, chiral (or triangle anomaly) is proportional to the trace of three hypercharges. In the SM one has

$$Tr Y^3 = \begin{array}{ccccccc} 3 & \left(\frac{1}{27} & +\frac{1}{27} & -\frac{64}{27} & +\frac{8}{27} \right) & -1 & -1 & +8 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ colour & u_L & d_L & u_R & d_R & \nu_L & e_L & e_R \end{array} = 0.$$

However, if one introduces a chiral Higgs superfield, it contains higgsinos, which are chiral fermions, and contain anomalies. To cancel them one has to add the second Higgs doublet with the opposite hypercharge.

Therefore, the Higgs sector in SUSY models is inevitably enlarged, it contains an even number of doublets.

Conclusion: In SUSY models supersymmetry associates *known* bosons with *new* fermions and *known* fermions with *new* bosons.

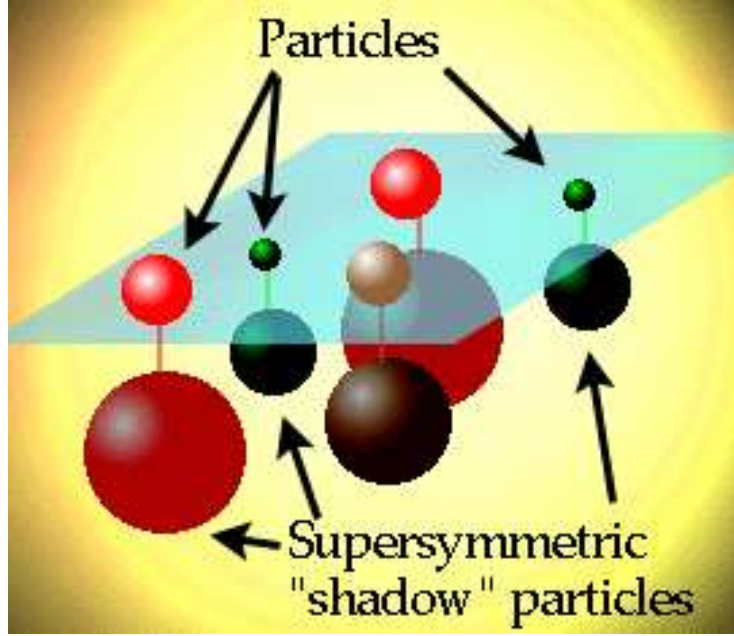


Figure 11: The shadow world of SUSY particles [26]

4.1 The field content

Consider the particle content of the Minimal Supersymmetric Standard Model [24]. According to the previous discussion, in the minimal version we double the number of particles (introducing a superpartner to each particle) and add another Higgs doublet (with its superpartner). The particle content of the MSSM then appears as [25]

Particle Content of the MSSM

Superfield	Bosons		Fermions		$SU_c(3)$	$SU_L(2)$	$U_Y(1)$
Gauge							
\mathbf{G}^a	gluon	g^a	gluino	\tilde{g}^a	8	0	0
\mathbf{V}^k	Weak	W^k (W^\pm, Z)	wino, zino	\tilde{w}^k (\tilde{w}^\pm, \tilde{z})	1	3	0
\mathbf{V}'	Hypercharge	B (γ)	bino	$\tilde{b}(\tilde{\gamma})$	1	1	0
Matter							
\mathbf{L}_i	sleptons	$\left\{ \begin{array}{l} \tilde{L}_i = (\tilde{\nu}, \tilde{e})_L \\ \tilde{E}_i = \tilde{e}_R \end{array} \right.$	leptons	$\left\{ \begin{array}{l} L_i = (\nu, e)_L \\ E_i = e_R \end{array} \right.$	1	2	-1
\mathbf{E}_i					1	1	2
\mathbf{Q}_i	squarks	$\left\{ \begin{array}{l} \tilde{Q}_i = (\tilde{u}, \tilde{d})_L \\ \tilde{U}_i = \tilde{u}_R \\ \tilde{D}_i = \tilde{d}_R \end{array} \right.$	quarks	$\left\{ \begin{array}{l} Q_i = (u, d)_L \\ U_i = u_R^c \\ D_i = d_R^c \end{array} \right.$	3	2	1/3
\mathbf{U}_i					3^*	1	-4/3
\mathbf{D}_i					3^*	1	2/3
Higgs							
\mathbf{H}_1	Higgses	$\left\{ \begin{array}{l} H_1 \\ H_2 \end{array} \right.$	higgsinos	$\left\{ \begin{array}{l} \tilde{H}_1 \\ \tilde{H}_2 \end{array} \right.$	1	2	-1
\mathbf{H}_2					1	2	1

where $a = 1, 2, \dots, 8$ and $k = 1, 2, 3$ are the $SU(3)$ and $SU(2)$ indices, respectively, and $i = 1, 2, 3$ is the generation index. Hereafter, tilde denotes a superpartner of an ordinary particle.

Thus, the characteristic feature of any supersymmetric generalization of the SM is the presence of superpartners (see Fig.11). If supersymmetry is exact, superpartners of ordinary particles should have the same masses and have to be observed. The absence of them at modern energies is believed to be explained by the fact that their masses are very heavy, that means that supersymmetry should be broken. Hence, if the energy of accelerators is high enough, the superpartners will be created.

The presence of an extra Higgs doublet in SUSY model is a novel feature of the theory. In the MSSM one has two doublets with the quantum numbers (1,2,-1) and (1,2,1), respectively:

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} v_1 + \frac{S_1 + iP_1}{H_1^- \sqrt{2}} \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} H_2^+ \\ v_2 + \frac{S_2 + iP_2}{\sqrt{2}} \end{pmatrix}, \quad (4.1)$$

where v_i are the vacuum expectation values of the neutral components.

Hence, one has $8=4+4=5+3$ degrees of freedom. As in the case of the SM, 3 degrees of freedom can be gauged away, and one is left with 5 physical states compared to 1 state in the SM.

Thus, in the MSSM, as actually in any of two Higgs doublet models, one has five physical Higgs bosons: two CP-even neutral, one CP-odd neutral and two charged. We consider the mass eigenstates below.

4.2 Lagrangian of the MSSM

The Lagrangian of the MSSM consists of two parts; the first part is SUSY generalization of the Standard Model, while the second one represents the SUSY breaking as mentioned above.

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{Breaking}, \quad (4.2)$$

where

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Gauge} + \mathcal{L}_{Yukawa} \quad (4.3)$$

and

$$\begin{aligned} \mathcal{L}_{Gauge} = & \sum_{SU(3), SU(2), U(1)} \frac{1}{4} \left(\int d^2\theta \text{Tr} W^\alpha W_\alpha + \int d^2\bar{\theta} \text{Tr} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \right) \\ & + \sum_{Matter} \int d^2\theta d^2\bar{\theta} \Phi_i^\dagger e^{g_3 \hat{V}_3 + g_2 \hat{V}_2 + g_1 \hat{V}_1} \Phi_i, \end{aligned} \quad (4.4)$$

$$\mathcal{L}_{Yukawa} = \int d^2\theta (\mathcal{W}_R + \mathcal{W}_{NR}) + h.c. \quad (4.5)$$

The index R in a superpotential refers to the so-called R -parity [27] which adjusts a "+" charge to all the ordinary particles and a "-" charge to their superpartners. The first part of \mathcal{W} is R-symmetric

$$W_R = \epsilon_{ij} (y_{ab}^U Q_a^j U_b^c H_2^i + y_{ab}^D Q_a^j D_b^c H_1^i + y_{ab}^L L_a^j E_b^c H_1^i + \mu H_1^i H_2^j), \quad (4.6)$$

where $i, j = 1, 2, 3$ are the $SU(2)$ and $a, b = 1, 2, 3$ are the generation indices; colour indices are suppressed. This part of the Lagrangian almost exactly repeats that of the SM except that the fields are now the superfields rather than the ordinary fields of the SM. The only difference is the last term which describes the Higgs mixing. It is absent in the SM since there is only one Higgs field there.

The second part is R -nonsymmetric

$$W_{NR} = \epsilon_{ij}(\lambda_{abd}^L L_a^i L_b^j E_d^c + \lambda_{abd}^{L'} L_a^i Q_b^j D_d^c + \mu_a' L_a^i H_2^j) + \lambda_{abd}^B U_a^c D_b^c D_d^c. \quad (4.7)$$

These terms are absent in the SM. The reason is very simple: one can not replace the superfields in eq.(4.7) by the ordinary fields like in eq.(4.6) because of the Lorentz invariance. These terms have a different property, they violate either lepton (the first line in eq.(4.7)) or baryon number (the second line). Since both effects are not observed in Nature, these terms must be suppressed or be excluded. One can avoid such terms if one introduces special symmetry called the R -symmetry [28]. This is the global $U(1)_R$ invariance

$$U(1)_R: \quad \theta \rightarrow e^{i\alpha}\theta, \quad \Phi \rightarrow e^{in\alpha}\Phi, \quad (4.8)$$

i.e., the superfield has the quantum number $R = n$. To preserve $U(1)_R$ invariance the superpotential W must have $R = 2$. Thus, to get $W_{NR} = 0$ one must choose $R = 1$ for all the Higgs superfields and $R = 1/2$ for quark and lepton ones. However, this property happens to be too restrictive. Indeed, the gaugino mass term, which is Lorentz and gauge invariant and is introduced while supersymmetry breaking, happens to be R -invariant only for $\alpha = \pm\pi$. This reduces the R -symmetry to the discrete group Z_2 , called the R -parity [27]. The R -parity quantum number is given by

$$R = (-1)^{3(B-L)+2S} \quad (4.9)$$

for particles with spin S . Thus, all the ordinary particles have the R -parity quantum number equal to $R = +1$, while all the superpartners have R -parity quantum number equal to $R = -1$. The R -parity obviously forbids the W_{NR} terms. It is usually assumed that they are absent in the MSSM, i.e. R -parity is preserved. However, there is no physical principle behind it. It may well be that these terms are present, though experimental limits on the couplings are very severe [29]

$$\lambda_{abc}^L, \quad \lambda_{abc}^{L'} < 10^{-4}, \quad \lambda_{abc}^B < 10^{-9}.$$

4.3 Properties of interactions

If one assumes that the R -parity is preserved, then the interactions of superpartners are essentially the same as in the SM, but two of three particles involved into an interaction at any vertex are replaced by superpartners. The reason for it, as we discussed earlier, is the R -parity. According to eq.(4.9), all the ordinary particles are R -even, while all the superpartners are R -odd.

Conservation of the R -parity has two consequences

- the superpartners are created in pairs;
- the lightest superparticle (LSP) is stable.

Usually it is photino $\tilde{\gamma}$, the superpartner of a photon with some admixture of neutral higgsino.

Typical vertices are shown in Figs.12-14. The tilde above a letter denotes the corresponding superpartner. Note that the coupling is the same in all the vertices involving superpartners.

In the case of R -parity violation one has additional vertices with new types of interaction. As has been already mentioned, they violate either the lepton or baryon number. The typical ones are

$$\mathcal{L}_{LLE} = \lambda' \{ \tilde{\nu}_L e_L e_R^c - \tilde{e}_L \nu_L e_R^c + \tilde{e}_R^* \nu_L e_R + \dots \}, \quad (4.10)$$

$$\mathcal{L}_{LQD} = \lambda \{ \tilde{\nu}_L d_L \bar{d}_R - \tilde{e}_L u_L \bar{d}_R + \tilde{d}_L \nu_L \bar{d}_R - \tilde{u}_L e_L \bar{d}_R + \dots \}. \quad (4.11)$$

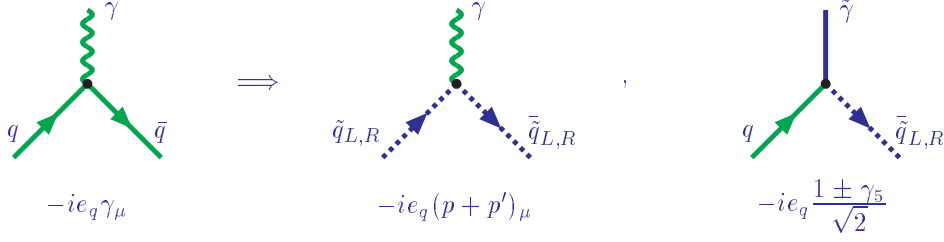


Figure 12: Gauge-matter interaction

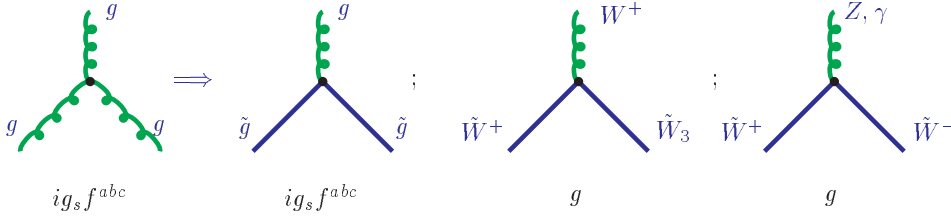


Figure 13: Gauge self-interaction

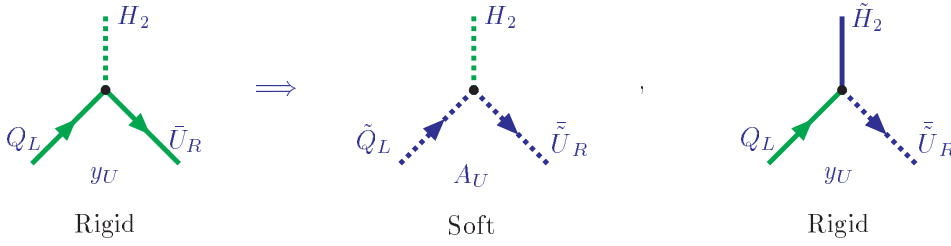


Figure 14: Yukawa-type interaction

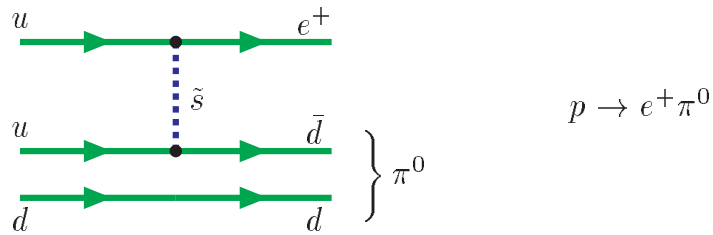


Figure 15: Proton decay in R-parity violating models

There are also UDD terms which violate the baryon number. These terms together lead to a fast proton decay via the process shown in Fig.15. To avoid it, one usually leaves either L or B violating interactions.

The limits on R -parity violating couplings come from non-observation of various processes, like proton decay, $\nu_\mu e$ scattering, etc and also from the charged current universality: $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu), \Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$, etc.

4.4 Creation and decay of superpartners

The above-mentioned rule together with the Feynman rules for the SM enables us to draw diagrams describing creation of superpartners. One of the most promising processes is the e^+e^- annihilation (see Fig.16). The usual kinematic restriction is given by the centre of mass energy

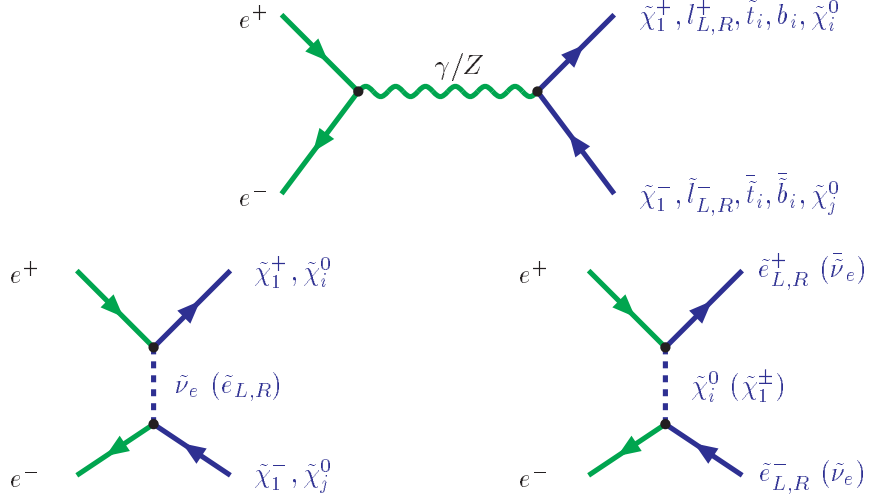


Figure 16: Creation of superpartners

$$m_{sparticle}^{max} \leq \frac{\sqrt{s}}{2}.$$

Similar processes take place at hadron colliders with electrons and positrons being replaced by quarks and gluons.

Creation of superpartners can be accompanied by creation of ordinary particles as well. We consider various experimental signatures for e^+e^- and hadron colliders below. They crucially depend on SUSY breaking pattern and on the mass spectrum of superpartners.

The decay properties of superpartners also depend on their masses. For the quark and lepton superpartners the main processes are shown in Fig.17.

When the R -parity is conserved, new particles will eventually end up giving neutralinos (the lightest superparticle) whose interactions are comparable to those of neutrinos and they leave undetected. Therefore, their signature would be missing energy and transverse momentum.

Examples. Consider some explicit examples of superpartner decays.

$$\begin{aligned}
\text{squarks : } \tilde{q}_{L,R} &\rightarrow q + \tilde{\chi}_i^0 && (\text{quark} + \text{photino}) \\
\tilde{q}_L &\rightarrow q' + \tilde{\chi}_i^\pm && (\text{quark} + \text{chargino}) \\
\tilde{q} &\rightarrow q + \tilde{g} && (\text{quark} + \text{gluino}) \text{ for } m_{\tilde{q}} > m_{\tilde{g}} \\
\tilde{t}_1 &\rightarrow c + \tilde{\chi}_1^0 && (\text{main decay}) \text{ signal: } 2 \text{ acollinear jets} + \cancel{E}_T \\
\tilde{t}_1 &\rightarrow b + \tilde{\chi}_1^+ && \text{signal: } 2 \text{ b jets} + 2 \text{ leptons} + \cancel{E}_T \\
&\hookrightarrow \tilde{\chi}_1^0 f \bar{f}' \quad (f \bar{f}' = l \bar{\nu}, q \bar{q}) && (4 \text{ jets}) + \cancel{E}_T
\end{aligned}$$

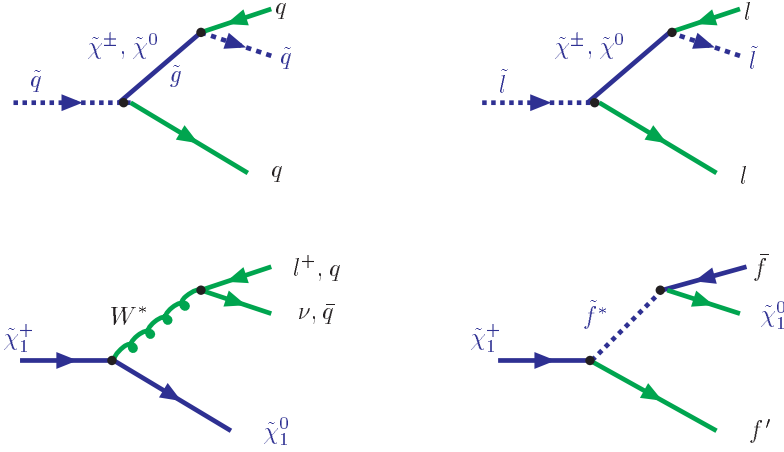


Figure 17: Decay of superpartners

sleptons :	$\tilde{l} \rightarrow l + \tilde{\chi}_i^0$	(lepton + photino)
	$\tilde{l}_L \rightarrow \nu_l + \tilde{\chi}_i^\pm$	(neutrino + chargino)
gluino :	$\tilde{g} \rightarrow q + \bar{q} + \tilde{\gamma}$	(quark + antiquark + photino)
	$\tilde{g} \rightarrow g + \tilde{\gamma}$	(gluon + photino)
chargino :	$\tilde{\chi}_i^\pm \rightarrow e + \nu_e + \tilde{\chi}_i^0$	(electron + neutrino + photino)
	$\tilde{\chi}_i^\pm \rightarrow q + \bar{q}' + \tilde{\chi}_i^0$	(quark + antiquark + photino)
neutralino :	$\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 + X$	

In the last case there are many possible channels both visible and invisible.

<u>Visible Channels</u>	<u>Final States</u>
$\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 l^+ l^- \quad (l = e, \mu, \tau)$	$l^+ l^- + \cancel{E}_T$
$\rightarrow \tilde{\chi}_1^\pm l^\mp \nu_l$	$l^\pm + \cancel{E}_T$
$\hookrightarrow \tilde{\chi}_1^0 l^\pm \nu_l$	
$\rightarrow \tilde{\chi}_1^0 q \bar{q}$	$2 \text{ jets} + \cancel{E}_T$
$\rightarrow \tilde{\chi}_\gamma^0$	$\gamma + \cancel{E}_T$
$\rightarrow \tilde{\chi}_1^\pm q \bar{q}'$	$2 \text{ jets} + \cancel{E}_T$
$\hookrightarrow \tilde{\chi}_1^0 l^\pm q \bar{q}'$	$l^\pm + 2 \text{ jets} + \cancel{E}_T$
$\rightarrow \tilde{\chi}_1^\pm q \bar{q}'$	$l^\pm + 2 \text{ jets} + \cancel{E}_T$
$\hookrightarrow \tilde{\chi}_1^0 l^\pm \nu_l$	
<u>Invisible Channel</u>	<u>Final State</u>
$\rightarrow \tilde{\chi}_1^0 \nu_l \bar{\nu}_l$	\cancel{E}_T

Thus, if supersymmetry exists in Nature and if it is broken somewhere below 1 TeV, then it will be possible to detect it in the nearest future.

5 Breaking of SUSY in the MSSM

Since none of the fields of the MSSM can develop non-zero v.e.v. to break SUSY without spoiling the gauge invariance, it is supposed that spontaneous supersymmetry breaking takes place via some other fields. The most common scenario for producing low-energy supersymmetry breaking is called the *hidden sector* one [30]. According to this scenario, there exist two sectors: the usual matter belongs to the "visible" one, while the second, "hidden" sector, contains fields which lead to breaking of supersymmetry. These two sectors interact with each other by exchange of some fields called *messengers*, which mediate SUSY breaking from the hidden to the visible sector (see Fig.18). There might be various types of messenger fields: gravity, gauge, etc. Below we consider four possible scenarios.

The hidden sector is the weakest part of the MSSM. It contains a lot of ambiguities and leads to uncertainties of the MSSM predictions considered below.

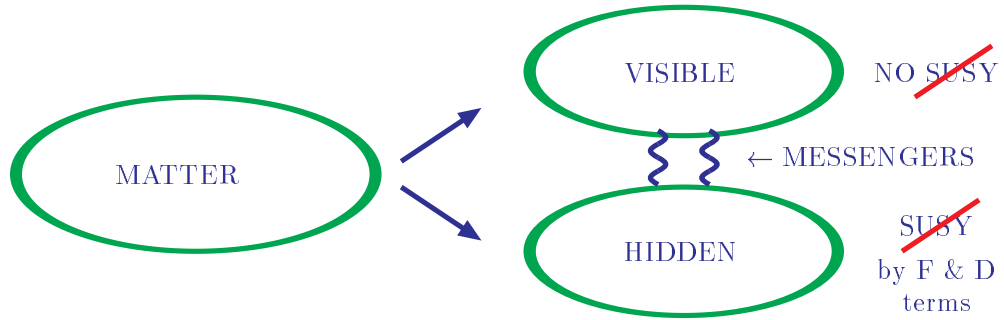


Figure 18: Hidden Sector Scenario

5.1 The hidden sector: four scenarios

So far there are known four main mechanisms to mediate SUSY breaking from a hidden to a visible sector:

- Gravity mediation (SUGRA);
- Gauge mediation;
- Anomaly mediation;
- Gaugino mediation.

Consider them in more detail.

SUGRA

This mechanism is based on effective nonrenormalizable interactions arising as a low-energy limit of supergravity theories [31]. In this case, two sectors interact with each other via gravity. There are two types of scalar fields that develop nonzero v.e.v.s, namely moduli fields T , which appear as a result of compactification from higher dimensions, and the dilaton field S , part of SUGRA supermultiplet. These fields obtain nonzero v.e.v.s for their F components: $\langle F_T \rangle \neq 0, \langle F_S \rangle \neq 0$, which leads to spontaneous SUSY breaking. Since in SUGRA theory supersymmetry is local, spontaneous breaking leads to Goldstone particle which is a Goldstone fermion in this case. With the help of a super-Higgs effect this particle may be absorbed into an additional component of a spin 3/2 particle, called *gravitino*, which becomes massive.

SUSY breaking is then mediated to a visible sector via gravitational interaction leading to the following SUSY breaking scale:

$$M_{SUSY} \sim \frac{\langle F_T \rangle}{M_{PL}} + \frac{\langle F_S \rangle}{M_{PL}} \sim m_{3/2},$$

where $m_{3/2}$ is the gravitino mass.

The effective low-energy theory, which emerges, contains explicit soft supersymmetry breaking terms

$$\mathcal{L}_{soft} = - \sum_i m_i^2 |A_i|^2 - \sum_i M_i (\lambda_i \lambda_i + \bar{\lambda}_i \bar{\lambda}_i) - \mathcal{B} \mathcal{W}^{(2)}(A) - \mathcal{A} \mathcal{W}^{(3)}(A), \quad (5.1)$$

where $\mathcal{W}^{(2)}$ and $\mathcal{W}^{(3)}$ are the quadratic and cubic terms of a superpotential, respectively. The mass parameters are

$$\begin{aligned} m_i^2 &\sim \left(\frac{\langle F_S \rangle}{M_{PL}} \right)^2 \sim m_{3/2}^2, & M_i &\sim \frac{\langle F_S \rangle}{M_{PL}} \sim m_{3/2}, \\ \mathcal{B} &\sim \left(\frac{\langle F_T \rangle}{M_{PL}} \right)^2 \sim m_{3/2}^2, & \mathcal{A} &\sim \frac{\langle F_{T,S} \rangle}{M_{PL}} \sim m_{3/2}. \end{aligned}$$

To have SUSY masses of an order of 1 TeV, one needs $\sqrt{\langle F_{T,S} \rangle} \sim 10^{11}$ GeV.

In spite of attractiveness of these mechanism in general, since we know that gravity exists anyway, it is not truly substantiated due to the lack of a consistent theory of quantum (super)gravity. Among the problems of a supergravity mechanism also are the large freedom of parameters and the absence of automatic suppression of flavour violation.

Gauge Mediation

In this version of a hidden sector scenario, the SUSY breaking effects are mediated to the observable world not via gravity but via gauge interactions [32]. The messengers are the gauge bosons and matter fields of the SM and of some GUT theory. The hidden sector is necessary since the dynamical SUSY breaking requires the fields with quantum numbers not compatible with the SM. The advantage of this scenario is that one can construct a renormalizable model with dynamic SUSY breaking, where in principle all the parameters can be calculated.

Consider some simplest possibility where in a hidden sector one has a singlet scalar superfield S with nonzero v.e.v. $\langle F_S \rangle \neq 0$. The messenger sector consists of some superfield Φ , for instance, $\bar{5}$ of SU(5), that couples to S and to the SM fields with a superpotential

$$\mathcal{W} \sim S \Phi^\dagger \Phi, \quad \langle S \rangle = M \neq 0. \quad (5.2)$$

Integrating out the messenger fields gives mass to gauginos at the one loop level (see Fig.19) and to the scalar fields (squarks and sleptons) at the two loop one (see Fig.20). So, in gauge mediated scenario all the soft masses are correlated to the gauge couplings and in this sense this scenario is more restrictive than the SUGRA one. There is no problem with flavour violating processes as well, since the soft terms automatically repeat the rigid sector.

It is remarkable that in this scenario the LSP happens to be the gravitino. The mass of the gravitino is given by

$$m_{\tilde{G}} \sim \frac{\langle F_S \rangle}{M} \cdot \frac{M}{M_{PL}} \sim 10^{-14} \frac{M}{[GeV]}, \quad (5.3)$$

that leads to a very light gravitino field.

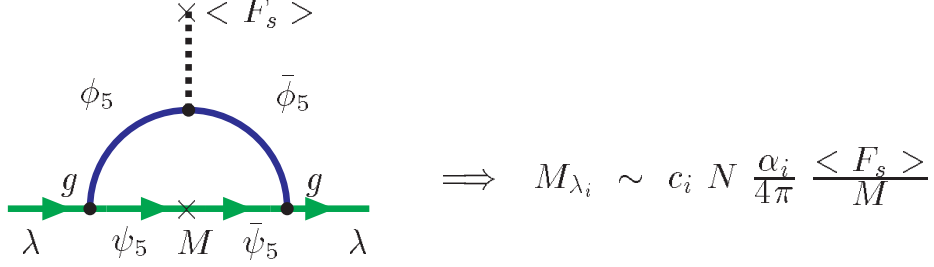


Figure 19: Gaugino mass generation

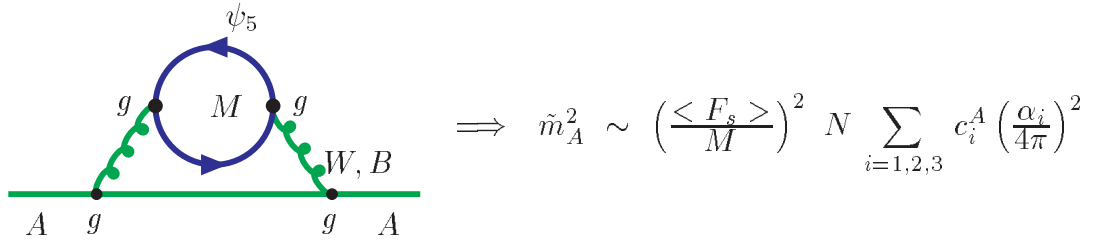


Figure 20: Squark mass generation

The problem of the gauge mediated SUSY breaking scenario emerges in the Higgs sector since the Higgs mass mixing parameters, which break an unwanted Peccei-Quin symmetry, cannot be generated by gauge interactions only. In order to parameterize some new unknown interactions, two new inputs have to be introduced (μ and B in SUGRA conventions).

Anomaly Mediation

An anomaly mediation mechanism assumes no SUSY breaking at the tree level. SUSY breaking is generated due to conformal anomaly. This mechanism refers to a hidden sector of a multidimensional theory with the couplings being dynamic fields which may acquire v.e.v.s. for their F components [33]. The external field or scale dependence of the couplings emerges as a result of conformal anomaly and that is why is proportional to the corresponding β functions. In the leading order one has

$$\begin{aligned} M_i(\Lambda) &\sim b_i \alpha_i(\Lambda) \frac{<F_{T,S}>}{M_{PL}} \sim b_i \alpha_i m_{3/2}, \\ m^2(\Lambda) &\sim b_i^2 \alpha_i^2(\Lambda) m_{3/2}^2, \end{aligned} \quad (5.4)$$

where b_i are the one-loop RG coefficients (see eq.(2.8)).

This reminds supergravity mediation mechanism but with fixed coefficients. It leads to two main differences:

i) the inverted relation between the gaugino masses at high energy scale

$$M_1 : M_2 : M_3 = b_1 : b_2 : b_3,$$

ii) negative slepton mass squared (tachyons!) at the tree level.

This problem has to be cured.

Gaugino Mediation

At last we would like to mention the gaugino mediation mechanism of SUSY breaking [34]. This is a less developed scenario so far. It is based on a paradigm of a brane world. According to this paradigm, there exists a multidimensional world where our four dimensional space-time represents a brane of 4 dimensions. The fields of the SM live on the brane, while gravity and some other fields can propagate in the bulk. There also exists another brane where supersymmetry is broken. SUSY breaking is mediated to our brane via the fields propagating in the bulk. It is assumed that the gaugino field plays an essential role in this mechanism (see Fig.21)

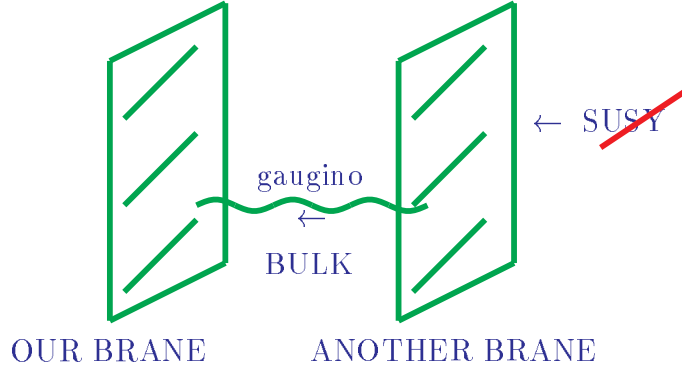


Figure 21: Gaugino mediated SUSY breaking

All four mechanisms of soft SUSY breaking are different in details but are common in results. They generate gauge invariant soft SUSY breaking operators of dimension ≤ 4 of the form

$$\begin{aligned} \mathcal{L}_{soft} = & - \sum_i m_i^2 |A_i|^2 - \sum_i M_i (\lambda_i \lambda_i + \bar{\lambda}_i \bar{\lambda}_i) \\ & - \sum_{ij} B_{ij} A_i A_j - \sum_{ijk} A_{ijk} A_i A_j A_k + h.c., \end{aligned} \quad (5.5)$$

where the bilinear and trilinear couplings B_{ij} and A_{ijk} are such that not to break the gauge invariance. These are the only possible soft terms that do not break renormalizability of a theory and preserve SUSY Ward identities for the rigid terms [35].

Predictions for the sparticle spectrum depend on the mechanism of SUSY breaking. For comparison of four above-mentioned mechanisms we show in Fig.22 the sample spectra as the ratio to the gaugino mass M_2 [36].

In what follows, to calculate the mass spectrum of superpartners, we need an explicit form of SUSY breaking terms. Applying eq.(5.5) to the MSSM and avoiding the R -parity violation gives

$$\begin{aligned} -\mathcal{L}_{Breaking} = & \sum_i m_{0i}^2 |\varphi_i|^2 + \left(\frac{1}{2} \sum_\alpha M_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha + B H_1 H_2 \right. \\ & \left. + A_{ab}^U \tilde{Q}_a \tilde{U}_b^c H_2 + A_{ab}^D \tilde{Q}_a \tilde{D}_b^c H_1 + A_{ab}^L \tilde{L}_a \tilde{E}_b^c H_1 + h.c. \right), \end{aligned} \quad (5.6)$$

where we have suppressed the $SU(2)$ indices. Here φ_i are all scalar fields, $\tilde{\lambda}_\alpha$ are the gaugino fields, $\tilde{Q}, \tilde{U}, \tilde{D}$ and \tilde{L}, \tilde{E} are the squark and slepton fields, respectively, and $H_{1,2}$ are the $SU(2)$ doublet Higgs fields.

SPARTICLE SPECTRA

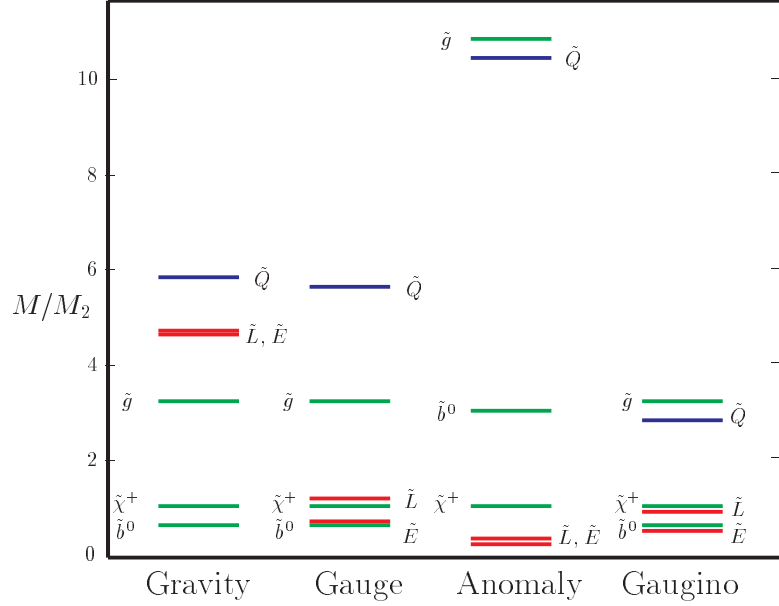


Figure 22: Superparticle spectra for various mediation mechanisms

Eq.(5.6) contains a vast number of free parameters which spoils the prediction power of the model. To reduce their number, we adopt the so-called universality hypothesis, i.e., we assume the universality or equality of various soft parameters at a high energy scale, namely, we put all the spin 0 particle masses to be equal to the universal value m_0 , all the spin 1/2 particle (gaugino) masses to be equal to $m_{1/2}$ and all the cubic and quadratic terms, proportional to A and B , to repeat the structure of the Yukawa superpotential (4.6). This is an additional requirement motivated by the supergravity mechanism of SUSY breaking. Universality is not a necessary requirement and one may consider nonuniversal soft terms as well. However, it will not change the qualitative picture presented below; so for simplicity, in what follows we consider the universal boundary conditions. In this case, eq.(5.6) takes the form

$$\begin{aligned}
 -\mathcal{L}_{Breaking} &= m_0^2 \sum_i |\varphi_i|^2 + \left(\frac{1}{2} m_{1/2} \sum_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha \right. \\
 &\quad \left. + A[y_{ab}^U \tilde{Q}_a \tilde{U}_b^c H_2 + y_{ab}^D \tilde{Q}_a \tilde{D}_b^c H_1 + y_{ab}^L \tilde{L}_a \tilde{E}_b^c H_1] + B[\mu H_1 H_2] + h.c. \right),
 \end{aligned} \tag{5.7}$$

It should be noted that supergravity induced universality of the soft terms is more likely to be valid at the Planck scale rather than at the GUT one. This is because a natural scale for gravity is M_{Planck} while M_{GUT} is the scale for gauge interactions. However, due to a small difference between these two scales, it is usually ignored in the first approximation resulting in minor uncertainties in the low-energy predictions [37].

The soft terms explicitly break supersymmetry. As will be shown later, they lead to the mass spectrum of superpartners different from that of ordinary particles. Remind that the masses of quarks and leptons remain zero until $SU(2)$ invariance is spontaneously broken.

5.2 The soft terms and the mass formulas

There are two main sources of the mass terms in the Lagrangian: the D terms and soft ones. With given values of $m_0, m_{1/2}, \mu, Y_t, Y_b, Y_\tau, A$, and B one can construct the mass matrices for all the particles. Knowing them at the GUT scale, one can solve the corresponding RG equations, thus linking the values at the GUT and electroweak scales. Substituting these parameters into the mass matrices, one can predict the mass spectrum of superpartners [24, 38, 39].

5.2.1 Gaugino-higgsino mass terms

The mass matrix for gauginos, the superpartners of the gauge bosons, and for higgsinos, the superpartners of the Higgs bosons, is nondiagonal, thus leading to their mixing. The mass terms look like

$$\mathcal{L}_{Gaugino-Higgsino} = -\frac{1}{2}M_3\bar{\lambda}_a\lambda_a - \frac{1}{2}\bar{\chi}M^{(0)}\chi - (\bar{\psi}M^{(c)}\psi + h.c.), \quad (5.8)$$

where $\lambda_a, a = 1, 2, \dots, 8$, are the Majorana gluino fields and

$$\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}, \quad \psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix} \quad (5.9)$$

are, respectively, the Majorana neutralino and Dirac chargino fields.

The neutralino mass matrix is

$$M^{(0)} = \begin{pmatrix} M_1 & 0 & -M_Z \cos \beta \sin_W & M_Z \sin \beta \sin_W \\ 0 & M_2 & M_Z \cos \beta \cos_W & -M_Z \sin \beta \cos_W \\ -M_Z \cos \beta \sin_W & M_Z \cos \beta \cos_W & 0 & -\mu \\ M_Z \sin \beta \sin_W & -M_Z \sin \beta \cos_W & -\mu & 0 \end{pmatrix}, \quad (5.10)$$

where $\tan \beta = v_2/v_1$ is the ratio of two Higgs v.e.v.s and $\sin_W = \sin \theta_W$ is the usual sinus of the weak mixing angle. The physical neutralino masses $M_{\tilde{\chi}_i^0}$ are obtained as eigenvalues of this matrix after diagonalization.

For charginos one has

$$M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu \end{pmatrix}. \quad (5.11)$$

This matrix has two chargino eigenstates $\tilde{\chi}_{1,2}^\pm$ with mass eigenvalues

$$M_{1,2}^2 = \frac{1}{2} \left[M_2^2 + \mu^2 + 2M_W^2 \mp \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2(M_2^2 + \mu^2 + 2M_2\mu \sin 2\beta)} \right]. \quad (5.12)$$

5.2.2 Squark and slepton masses

Non-negligible Yukawa couplings cause a mixing between the electroweak eigenstates and the mass eigenstates of the third generation particles. The mixing matrices for $\tilde{m}_t^2, \tilde{m}_b^2$ and \tilde{m}_τ^2 are

$$\begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu \cot \beta) \\ m_t(A_t - \mu \cot \beta) & \tilde{m}_{tR}^2 \end{pmatrix}, \quad (5.13)$$

$$\begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b - \mu \tan \beta) \\ m_b(A_b - \mu \tan \beta) & \tilde{m}_{bR}^2 \end{pmatrix}, \quad (5.14)$$

$$\begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau - \mu \tan \beta) \\ m_\tau(A_\tau - \mu \tan \beta) & \tilde{m}_{\tau R}^2 \end{pmatrix} \quad (5.15)$$

with

$$\begin{aligned} \tilde{m}_{tL}^2 &= \tilde{m}_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2) \cos 2\beta, \\ \tilde{m}_{tR}^2 &= \tilde{m}_U^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2) \cos 2\beta, \\ \tilde{m}_{bL}^2 &= \tilde{m}_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2) \cos 2\beta, \\ \tilde{m}_{bR}^2 &= \tilde{m}_D^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2) \cos 2\beta, \\ \tilde{m}_{\tau L}^2 &= \tilde{m}_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2) \cos 2\beta, \\ \tilde{m}_{\tau R}^2 &= \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2) \cos 2\beta \end{aligned}$$

and the mass eigenstates are the eigenvalues of these mass matrices. For the light generations the mixing is negligible.

The first terms here (\tilde{m}^2) are the soft ones, which are calculated using the RG equations starting from their values at the GUT (Planck) scale. The second ones are the usual masses of quarks and leptons and the last ones are the D terms of the potential.

5.3 The Higgs potential

As has already been mentioned, the Higgs potential in the MSSM is totally defined by superpotential (and the soft terms). Due to the structure of \mathcal{W} the Higgs self-interaction is given by the D -terms while the F -terms contribute only to the mass matrix. The tree level potential is

$$\begin{aligned} V_{tree}(H_1, H_2) &= m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.) \\ &+ \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} |H_1^+ H_2|^2, \end{aligned} \quad (5.16)$$

where $m_1^2 = m_{H_1}^2 + \mu^2$, $m_2^2 = m_{H_2}^2 + \mu^2$. At the GUT scale $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$, $m_3^2 = -B\mu_0$. Notice that the Higgs self-interaction coupling in eq.(5.16) is fixed and defined by the gauge interactions as opposed to the SM.

The potential (5.16), in accordance with supersymmetry, is positive definite and stable. It has no nontrivial minimum different from zero. Indeed, let us write the minimization condition for the potential (5.16)

$$\frac{1}{2} \frac{\delta V}{\delta H_1} = m_1^2 v_1 - m_3^2 v_2 + \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) v_1 = 0, \quad (5.17)$$

$$\frac{1}{2} \frac{\delta V}{\delta H_2} = m_2^2 v_2 - m_3^2 v_1 + \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) v_2 = 0, \quad (5.18)$$

where we have introduced the notation

$$\langle H_1 \rangle \equiv v_1 = v \cos \beta, \quad \langle H_2 \rangle \equiv v_2 = v \sin \beta, \quad v^2 = v_1^2 + v_2^2, \quad \tan \beta \equiv \frac{v_2}{v_1}.$$

Solution of eqs.(5.17),(5.18) can be expressed in terms of v^2 and $\sin 2\beta$

$$v^2 = \frac{4(m_1^2 - m_2^2 \tan^2 \beta)}{(g^2 + g'^2)(\tan^2 \beta - 1)}, \quad \sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}. \quad (5.19)$$

One can easily see from eq.(5.19) that if $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$, v^2 happens to be negative, i.e. the minimum does not exist. In fact, real positive solutions to eqs.(5.17),(5.18) exist only if the following conditions are satisfied [25]:

$$m_1^2 + m_2^2 > 2m_3^2, \quad m_1^2 m_2^2 < m_3^4, \quad (5.20)$$

which is not the case at the GUT scale. This means that spontaneous breaking of the $SU(2)$ gauge invariance, which is needed in the SM to give masses for all the particles, does not take place in the MSSM.

This strong statement is valid, however, only at the GUT scale. Indeed, going down with energy, the parameters of the potential (5.16) are renormalized. They become the “running” parameters with the energy scale dependence given by the RG equations. The running of the parameters leads to a remarkable phenomenon known as *radiative spontaneous symmetry breaking* to be discussed below.

Provided conditions (5.20) are satisfied, the mass matrices at the tree level are CP-odd components P_1 and P_2 :

$$\mathcal{M}^{odd} = \frac{\partial^2 V}{\partial P_i \partial P_j} \Big|_{H_i=v_i} = \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix} m_3^2, \quad (5.21)$$

CP-even neutral components S_1 and S_2 :

$$\mathcal{M}^{even} = \frac{\partial^2 V}{\partial S_i \partial S_j} \Big|_{H_i=v_i} = \begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} m_3^2 + \begin{pmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{pmatrix} M_Z \cos \beta \sin \beta, \quad (5.22)$$

Charged components H^- and H^+ :

$$\mathcal{M}^{charged} = \frac{\partial^2 V}{\partial H_i^+ \partial H_j^-} \Big|_{H_i=v_i} = \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix} (m_3^2 + M_W \cos \beta \sin \beta). \quad (5.23)$$

Diagonalizing the mass matrices, one gets the mass eigenstates [25]:

$$\begin{cases} G^0 &= -\cos \beta P_1 + \sin \beta P_2, & \text{Goldstone boson} \rightarrow Z_0, \\ A &= \sin \beta P_1 + \cos \beta P_2, & \text{Neutral CP} = -1 \text{ Higgs}, \end{cases}$$

$$\begin{cases} G^+ &= -\cos \beta (H_1^-)^* + \sin \beta H_2^+, & \text{Goldstone boson} \rightarrow W^+, \\ H^+ &= \sin \beta (H_1^-)^* + \cos \beta H_2^+, & \text{Charged Higgs}, \end{cases}$$

$$\begin{cases} h &= -\sin \alpha S_1 + \cos \alpha S_2, & \text{SM Higgs boson CP} = 1, \\ H &= \cos \alpha S_1 + \sin \alpha S_2, & \text{Extra heavy Higgs boson}, \end{cases}$$

where the mixing angle α is given by

$$\tan 2\alpha = -\tan 2\beta \left(\frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2} \right).$$

The physical Higgs bosons acquire the following masses [24]:

$$\begin{aligned} \text{CP-odd neutral Higgs } A : & \quad m_A^2 = m_1^2 + m_2^2, \\ \text{Charge Higgses } H^\pm : & \quad m_{H^\pm}^2 = m_A^2 + M_W^2, \end{aligned} \quad (5.24)$$

CP-even neutral Higgses H, h :

$$m_{H,h}^2 = \frac{1}{2} \left[m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \right], \quad (5.25)$$

where, as usual,

$$M_W^2 = \frac{g^2}{2} v^2, \quad M_Z^2 = \frac{g^2 + g'^2}{2} v^2.$$

This leads to the once celebrated SUSY mass relations

$$\begin{aligned} m_{H^\pm} &\geq M_W, \\ m_h &\leq m_A \leq M_H, \\ m_h &\leq M_Z |\cos 2\beta| \leq M_Z, \\ m_h^2 + m_H^2 &= m_A^2 + M_Z^2. \end{aligned} \quad (5.26)$$

Thus, the lightest neutral Higgs boson happens to be lighter than the Z boson, which clearly distinguishes it from the SM one. Though we do not know the mass of the Higgs boson in the SM, there are several indirect constraints leading to the lower boundary of $m_h^{SM} \geq 135$ GeV [40]. After including the radiative corrections, the mass of the lightest Higgs boson in the MSSM, m_h , however increases. We consider it in more detail below.

5.4 Renormalization group analysis

To calculate the low energy values of the soft terms, we use the corresponding RG equations. The one-loop RG equations for the rigid MSSM couplings are [41]

$$\begin{aligned} \frac{d\tilde{\alpha}_i}{dt} &= b_i \tilde{\alpha}_i^2, \quad t \equiv \log Q^2 / M_{GUT}^2 \\ \frac{dY_U}{dt} &= -Y_L \left(\frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{15} \tilde{\alpha}_1 - 6Y_U - Y_D \right), \\ \frac{dY_D}{dt} &= -Y_D \left(\frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{7}{15} \tilde{\alpha}_1 - Y_U - 6Y_D - Y_L \right), \\ \frac{dY_L}{dt} &= -Y_L \left(3\tilde{\alpha}_2 + \frac{9}{5} \tilde{\alpha}_1 - 3Y_D - 4Y_L \right), \end{aligned} \quad (5.27)$$

where we use the notation $\tilde{\alpha} = \alpha/4\pi = g^2/16\pi^2$, $Y = y^2/16\pi^2$.

For the soft terms one finds

$$\begin{aligned} \frac{dM_i}{dt} &= b_i \tilde{\alpha}_i M_i. \\ \frac{dA_U}{dt} &= \frac{16}{3} \tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{13}{15} \tilde{\alpha}_1 M_1 + 6Y_U A_U + Y_D A_D, \\ \frac{dA_D}{dt} &= \frac{16}{3} \tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{7}{15} \tilde{\alpha}_1 M_1 + 6Y_D A_D + Y_U A_U + Y_L A_L, \\ \frac{dA_L}{dt} &= 3\tilde{\alpha}_2 M_2 + \frac{9}{5} \tilde{\alpha}_1 M_1 + 3Y_D A_D + 4Y_L A_L, \end{aligned}$$

$$\begin{aligned}
\frac{dB}{dt} &= 3\tilde{\alpha}_2 M_2 + \frac{3}{5}\tilde{\alpha}_1 M_1 + 3Y_U A_U + 3Y_D A_D + Y_L A_L. \\
\frac{d\tilde{m}_Q^2}{dt} &= - \left[\left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \frac{1}{15}\tilde{\alpha}_1 M_1^2 \right) - Y_U(\tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_U^2) \right. \\
&\quad \left. - Y_D(\tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2 + A_D^2) \right], \\
\frac{d\tilde{m}_U^2}{dt} &= - \left[\left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{16}{15}\tilde{\alpha}_1 M_1^2 \right) - 2Y_U(\tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_U^2) \right], \\
\frac{d\tilde{m}_D^2}{dt} &= - \left[\left(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{4}{15}\tilde{\alpha}_1 M_1^2 \right) - 2Y_D(\tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2 + A_D^2) \right], \\
\frac{d\tilde{m}_L^2}{dt} &= - \left[3(\tilde{\alpha}_2 M_2^2 + \frac{1}{5}\tilde{\alpha}_1 M_1^2) - Y_L(\tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2 + A_L^2) \right], \\
\frac{d\tilde{m}_E^2}{dt} &= - \left[\left(\frac{12}{5}\tilde{\alpha}_1 M_1^2 \right) - 2Y_L(\tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2 + A_L^2) \right], \\
\frac{d\mu^2}{dt} &= -\mu^2 \left[3(\tilde{\alpha}_2 + \frac{1}{5}\tilde{\alpha}_1) - (3Y_U + 3Y_D + Y_L) \right], \\
\frac{dm_{H_1}^2}{dt} &= - \left[3(a_2 M_2^2 + \frac{1}{5}a_1 M_1^2) - 3Y_D(\tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2 + A_D^2) \right. \\
&\quad \left. - Y_L(\tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2 + A_L^2) \right], \\
\frac{dm_{H_2}^2}{dt} &= - \left[3(a_2 M_2^2 + \frac{1}{5}a_1 M_1^2) - 3Y_U(\tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_U^2) \right].
\end{aligned} \tag{5.28}$$

Having all the RG equations, one can now find the RG flow for the soft terms. To see what happens at lower scales, one has to run the RG equations for the mass parameters in the opposite direction from the GUT to the EW scale. Let us take some initial values of the soft masses at the GUT scale in the interval between $10^2 \div 10^3$ GeV consistent with the SUSY scale suggested by unification of the gauge couplings (2.9). This leads to the following RG flow of the soft terms shown in Fig.23. [38, 39]

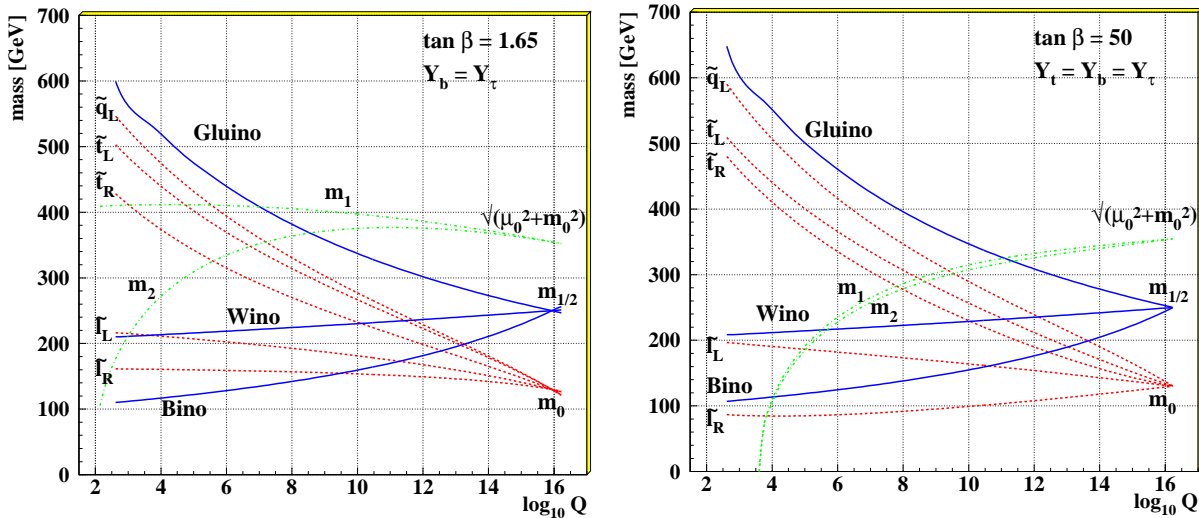


Figure 23: An example of evolution of particle masses and soft supersymmetry breaking parameters $m_1^2 = m_{H_1}^2 + \mu^2$ and $m_2^2 = m_{H_2}^2 + \mu^2$ for low (left) and high (right) values of $\tan \beta$

One should mention the following general features common to any choice of initial conditions:

i) The gaugino masses follow the running of the gauge couplings and split at low energies. The gluino mass is running faster than the others and is usually the heaviest due to the strong interaction.

ii) The squark and slepton masses also split at low energies, the stops (and sbottoms) being the lightest due to relatively big Yukawa couplings of the third generation.

iii) The Higgs masses (or at least one of them) are running down very quickly and may even become negative.

To calculate the masses one has also to take into account the mixing between various states (see eqs.(5.10,5.11, 5.13-5.15)).

Numerical solutions allow one to understand the significance of different initial conditions for the evolution down to low energies. As an example we present below the results of a numerical solution to the RG equations for the soft terms in the case of low values of $\tan\beta$. In this case, one can ignore the bottom and tau Yukawa couplings and keep only the top one. Taking $M_{GUT} = 2.0 \cdot 10^{16}$ GeV, $\alpha(M_{GUT}) \approx 1/24.3$, $Y_t(M_{GUT}) \approx \tilde{\alpha}(M_{GUT})$, $\tan\beta = 1.65$, one gets the following numerical results [39]:

$$\begin{aligned}
M_3(M_Z) &= 2.7 m_{1/2}, \\
M_2(M_Z) &= 0.8 m_{1/2}, \\
M_1(M_Z) &= 0.4 m_{1/2}, \\
\mu(M_Z) &= 0.63 \mu_0, \\
A_t(M_Z) &= 0.009 A_t(0) - 1.7 m_{1/2}, \\
\tilde{m}_{E_L}^2(M_Z) &= m_0^2 + 0.52 m_{1/2}^2 - 0.27 \cos(2\beta) M_Z^2, \\
\tilde{m}_{\nu_L}^2(M_Z) &= m_0^2 + 0.52 m_{1/2}^2 + 0.5 \cos(2\beta) M_Z^2, \\
\tilde{m}_{E_R}^2(M_Z) &= m_0^2 + 0.15 m_{1/2}^2 - 0.23 \cos(2\beta) M_Z^2, \\
\tilde{m}_{U_L}^2(M_Z) &= m_0^2 + 6.6 m_{1/2}^2 + 0.35 \cos(2\beta) M_Z^2, \\
\tilde{m}_{D_L}^2(M_Z) &= m_0^2 + 6.6 m_{1/2}^2 - 0.42 \cos(2\beta) M_Z^2, \\
\tilde{m}_{U_R}^2(M_Z) &= m_0^2 + 6.2 m_{1/2}^2 + 0.15 \cos(2\beta) M_Z^2, \\
\tilde{m}_{D_R}^2(M_Z) &= m_0^2 + 6.1 m_{1/2}^2 - 0.07 \cos(2\beta) M_Z^2, \\
\tilde{m}_{b_R}^2(M_Z) &= \tilde{m}_{D_R}^2, \\
\tilde{m}_{b_L}^2(M_Z) &= \tilde{m}_{D_L}^2 - 0.48 m_0^2 - 1.21 m_{1/2}^2, \\
\tilde{m}_{t_R}^2(M_Z) &= \tilde{m}_{U_R}^2 - 0.96 m_0^2 - 2.42 m_{1/2}^2, \\
\tilde{m}_{t_L}^2(M_Z) &= \tilde{m}_{U_L}^2 - 0.48 m_0^2 - 1.21 m_{1/2}^2, \\
m_1^2(M_Z) &= m_0^2 + 0.40 \mu_0^2 + 0.52 m_{1/2}^2, \\
m_2^2(M_Z) &= -0.44 m_0^2 + 0.40 \mu_0^2 - 3.11 m_{1/2}^2 - 0.09 A_0 m_{1/2} - 0.02 A_0^2.
\end{aligned}$$

Typical dependence of the mass spectra on the initial conditions (m_0) is also shown in Fig.24 [42]. For a given value of $m_{1/2}$ the masses of the lightest particles are practically independent of m_0 , while the heavier ones increase with it monotonically as it follows also from the numerical solutions given above. One can see that the lightest neutralinos and charginos as well as the stop squark may be rather light.

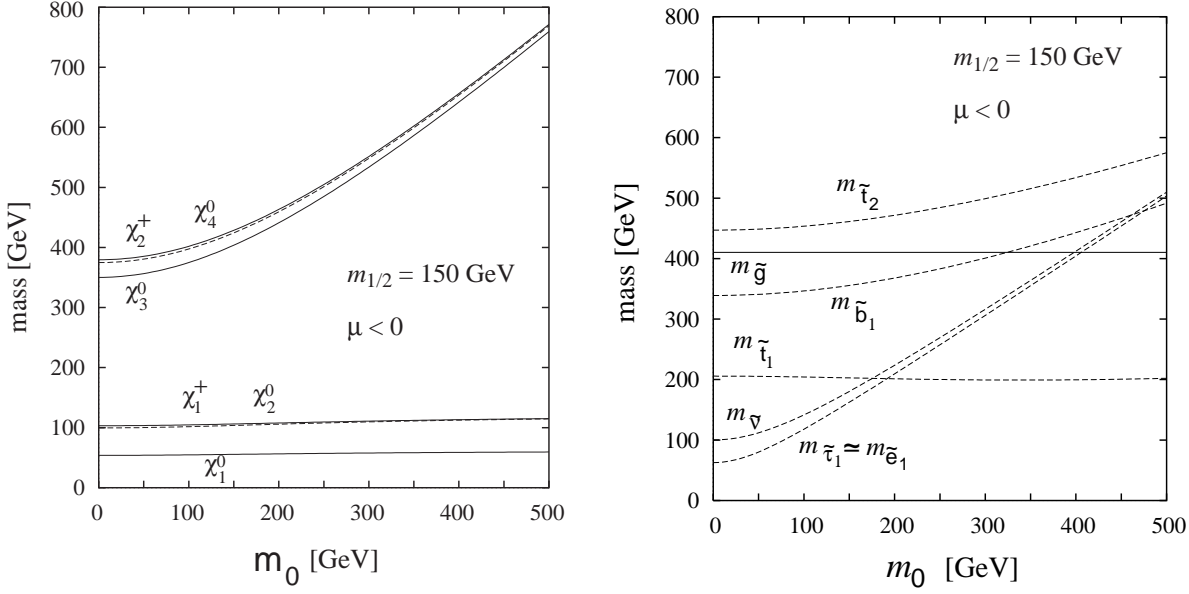


Figure 24: The masses of particles as functions of the initial value m_0

5.5 Radiative electroweak symmetry breaking

The running of the Higgs masses leads to the phenomenon known as *radiative electroweak symmetry breaking*. By this we mean the following: at the GUT energy scale both the Higgs mass parameters m_1^2 and m_2^2 are positive, and the Higgs potential has no nontrivial minima. However, when running down to the EW scale due to the radiative corrections they may change the sign so that the potential develops a nontrivial minimum. At this minimum the electroweak symmetry happens to be spontaneously broken. Thus, contrary to the SM, where one has to choose the negative sign of the Higgs mass squared "by hand", in the MSSM the effect of spontaneous symmetry breaking is triggered by the radiative corrections.

Indeed, one can see in Fig.23 that m_2^2 (or both m_1^2 and m_2^2) decreases when going down from the GUT scale to the M_Z scale and can even become negative. This is the effect of the large top (and bottom) Yukawa couplings in the RG equations. As a result, at some value of Q^2 the conditions (5.20) are satisfied, so that the nontrivial minimum appears. This triggers spontaneous breaking of the $SU(2)$ gauge invariance. The vacuum expectations of the Higgs fields acquire nonzero values and provide masses to quarks, leptons and $SU(2)$ gauge bosons, and additional masses to their superpartners.

In this way one also obtains the explanation of why the two scales are so much different. Due to the logarithmic running of the parameters, one needs a long "running time" to get m_2^2 (or both m_1^2 and m_2^2) to be negative when starting from a positive value of the order of $M_{SUSY} \sim 10^2 \div 10^3$ GeV at the GUT scale.

6 Constrained MSSM

6.1 Parameter space of the MSSM

The Minimal Supersymmetric Standard Model has the following free parameters:

- Three gauge couplings α_i .

- The matrices of the Yukawa couplings y_{ab}^i , where $i = L, U, D$.
- The Higgs field mixing parameter μ .
- The soft supersymmetry breaking parameters.

Compared to the SM there is an additional Higgs mixing parameter, but the Higgs self-coupling, which is arbitrary in the SM, is fixed by supersymmetry. The main uncertainty comes from the unknown soft terms.

With universality hypothesis one is left with the following set of 5 free parameters defining the mass scales

$$\mu, m_0, m_{1/2}, A \text{ and } B.$$

Parameter B is usually traded for $\tan \beta$, the ratio of the v.e.v.s of the two Higgs fields.

In particular models, like in SUGRA or gauge and anomaly mediation, some of soft parameters may be related to each other. However, since the mechanism of SUSY breaking is unknown, in what follows we consider them as free phenomenological parameters to be fitted by experiment. The experimental constraints are sufficient to determine these parameters, albeit with large uncertainties. The statistical analysis yields the probability for every point in the SUSY parameter space, which allows one to calculate the cross sections for the expected new physics of the MSSM at the existing or future accelerators (LEP II, Tevatron, LHC).

While choosing parameters and making predictions, one has two possible ways to proceed:

- i) take the low-energy parameters as input, impose the constraints, define the allowed parameter space and calculate the spectrum and cross-sections as functions of these parameters. They might be the superparticle masses $\tilde{m}_{t1}, \tilde{m}_{t2}, m_A, \tan \beta$, mixings X_{stop}, μ , etc.
- ii) take the high-energy parameters as input, run the RG equations, find the low-energy values, then impose the constraints and define the allowed parameter space for initial values. Now the calculations can be carried out in terms of the initial parameters. They might be, for example, the above mentioned 5 soft parameters.

Both the ways are used in a phenomenological analysis. We show below how it works in practice.

6.2 The choice of constraints

Among the constraints that we are going to impose on the MSSM model are those which follow from the comparison of the SM with experimental data, from the experimental limits on the masses of as yet unobserved particles, etc, and also those that follow from the ideas of unification and from SUSY GUT models. Some of them look very obvious while the others depend on a choice. Perhaps, the most remarkable fact is that all of them can be fulfilled simultaneously. The only model where one can do it is proved to be the MSSM.

In our analysis we impose the following constraints on the parameter space of the MSSM:

- Gauge coupling constant unification;

This is one of the most restrictive constraints, which we have discussed in Sect 2. It fixes the scale of SUSY breaking of an order of 1 TeV.

- M_Z from electroweak symmetry breaking;

Radiative corrections trigger spontaneous symmetry breaking in the electroweak sector. In this case, the Higgs potential does not have its minimum for all fields equal to zero, but the minimum is obtained for nonzero vacuum expectation values of the fields. Solving M_Z from eq.(5.19) yields

$$M_Z^2 = 2 \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1}. \quad (6.1)$$

To get the right value of M_Z requires proper adjustment of parameters. This condition determines the value of μ for given values of m_0 and $m_{1/2}$.

- Yukawa coupling constant unification;

The masses of top, bottom and τ can be obtained from the low energy values of the running Yukawa couplings

$$m_t = y_t v \sin \beta, \quad m_b = y_b v \cos \beta, \quad m_\tau = y_\tau v \cos \beta. \quad (6.2)$$

Eq.(6.2) is written for the so-called running masses. They can be translated to the pole masses with account taken of the radiative corrections. For the pole masses of the third generation the following values are taken [43], [1]

$$\begin{aligned} M_t &= 174.3 \pm 5.1 \text{ GeV}/c^2, \\ M_b &= 4.94 \pm 0.15 \text{ GeV}/c^2, \\ M_\tau &= 1.7771 \pm 0.0005 \text{ GeV}/c^2. \end{aligned} \quad (6.3)$$

The requirement of bottom-tau Yukawa coupling unification strongly restricts the possible solutions in m_t versus $\tan \beta$ plane [44]-[49] as it can be seen from Fig.25.

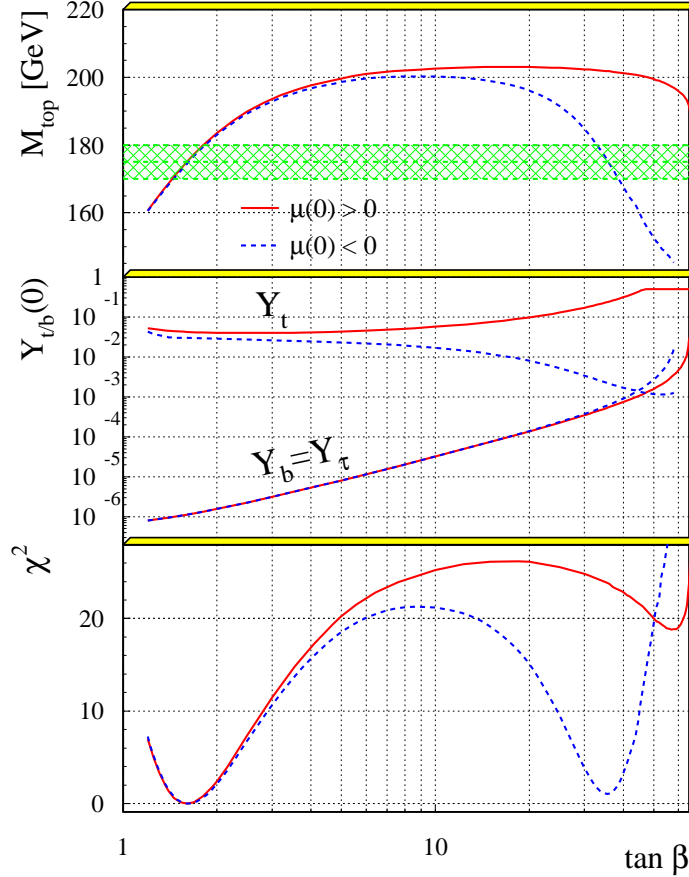


Figure 25: The upper part shows the top quark mass as a function of $\tan \beta$ for $m_0 = 600$ GeV, $m_{1/2} = 400$ GeV. The middle part shows the corresponding values of the Yukawa couplings at the GUT scale and the lower part of the χ^2 values.

- Branching ratio $BR(b \rightarrow s\gamma)$;

The branching ratio $BR(b \rightarrow s\gamma)$ has been measured by the CLEO [50] collaboration and later by ALEPH [51] and yields the world average of $BR(b \rightarrow s\gamma) = (3.14 \pm 0.48) \cdot 10^{-4}$. The Standard Model contribution to this process comes from the $W - t$ loop and gives a prediction which is very close to the experimental value leaving little space for SUSY. In the MSSM, this flavour changing neutral current (FCNC) receives additional contributions from the $H^\pm - t$, $\tilde{\chi}^\pm - \tilde{t}$ and $\tilde{g} - \tilde{q}$ loops. The $\tilde{\chi}^0 - \tilde{t}$ loops are much smaller [52, 53]. In the leading order, SUSY contribution may be rather big, exceeding the experimental value by several standard deviations. However, the NLO corrections are essential.

This requirement imposes severe restrictions on the parameter space, especially for the case of large $\tan \beta$.

- Experimental lower limits on SUSY masses;

SUSY particles have not been found so far and from the searches at LEP one knows the lower limit on the charged lepton and chargino masses of about half of the centre of mass energy [54]. The lower limit on the neutralino masses is smaller. The lower limit on the Higgs mass is roughly given by the c.m.e. minus the Z-boson mass. These limits restrict the minimal values for the SUSY mass parameters. There exist also limits on squark and gluino masses from the hadron colliders [55], but these limits depend on the assumed decay modes. Furthermore, if one takes the limits given above into account, the constraints from the limits on all other particles are usually fulfilled, so they do not provide additional reductions of the parameter space in the case of the minimal SUSY model.

- Dark Matter constraint;

Abundant evidence of the existence of nonrelativistic, neutral, nonbaryonic dark matter exists in our Universe [56, 57]. The lightest supersymmetric particle (LSP) is supposedly stable and would be an ideal candidate for dark matter.

The present lifetime of the universe is at least 10^{10} years, which implies an upper limit on the expansion rate and correspondingly on the total relic abundance. Assuming $h_0 > 0.4$ one finds that the contribution of each relic particle species χ has to obey [57]

$$\Omega_\chi h_0^2 < 1,$$

where $\Omega_\chi h^2$ is the ratio of the relic particle density of particle χ and the critical density, which overcloses the Universe. This bound can only be met, if most of the LSP's annihilated into fermion-antifermion pairs, which in turn would annihilate into photons again.

Since the neutralinos are mixtures of gauginos and higgsinos, the annihilation can occur both, via s-channel exchange of the Z^0 and Higgs bosons and t-channel exchange of a scalar particle, like a selectron [58]. This constrains the parameter space, as discussed by many groups [59]-[62].

- Proton life-time constraint;

There are two sources of proton decay in SUSY GUTs. The first one is the same as in non-SUSY theories and is related to the s-channel exchange of heavy gauge bosons. To avoid contradiction with experiment, the unification scale has to be above 10^{15} GeV which is usually satisfied in any SUSY GUT.

The second source is more specific to SUSY models. The proton decay in this case takes place due to the loop diagrams with the exchange of heavy higgsino triplets. The preferable decay mode in this case is $p \rightarrow \bar{\nu} K$ or $p \rightarrow \mu^+ K$ instead of $p \rightarrow e^+ \pi$ in non-SUSY GUTs. The decay rate in this case depends on a particular GUT model and it is not so easy to satisfy the experimental requirements.

Having in mind the above mentioned constraints one can try to fix the arbitrariness in the parameters. In a kind of a statistical analysis, in which all the constraints are implemented in a

χ^2 definition, one can find the most probable region of the parameter space by minimizing the χ^2 function. For the purpose of this analysis the following χ^2 definition is used [39]:

$$\begin{aligned}
\chi^2 = & \sum_{i=1}^3 \frac{(\alpha_i^{-1}(M_Z) - \alpha_{MSSM_i}^{-1}(M_Z))^2}{\sigma_i^2} \\
& + \frac{(M_Z - 91.18)^2}{\sigma_Z^2} + \frac{(M_t - 174)^2}{\sigma_t^2} \\
& + \frac{(M_b - 4.94)^2}{\sigma_b^2} + \frac{(M_\tau - 1.7771)^2}{\sigma_\tau^2} \\
& + \frac{(Br(b \rightarrow s\gamma) - 3.14 \times 10^{-4})^2}{\sigma(b \rightarrow s\gamma)^2} \\
& + \frac{(\Omega h^2 - 1)^2}{\sigma_\Omega^2} \quad (for \ \Omega h^2 > 1) \\
& + \frac{(\tilde{M} - \tilde{M}_{exp})^2}{\sigma_{\tilde{M}}^2} \quad (for \ \tilde{M} < \tilde{M}_{exp}) \\
& + \frac{(\tilde{m}_{LSP} - \tilde{m}_\chi)^2}{\sigma_{LSP}^2} \quad (for \ \tilde{m}_{LSP} \text{ charged}).
\end{aligned} \tag{6.4}$$

The first six terms are used to enforce gauge coupling unification, electroweak symmetry breaking and $b - \tau$ Yukawa coupling unification, respectively. The following two terms impose the constraints from $b \rightarrow s\gamma$ and the relic density, while the last terms require the SUSY masses to be above the experimental lower limits and the lightest supersymmetric particle (LSP) to be a neutralino since a charged stable LSP would have been observed. The input and fitted output variables have been summarized in Table 1.

exp. input data	\Rightarrow	Fit parameters	
		low $\tan \beta$	high $\tan \beta$
$\alpha_1, \alpha_2, \alpha_3$	minimize χ^2	M_{GUT}, α_{GUT}	M_{GUT}, α_{GUT}
m_t		$Y_t^0, Y_b^0 = Y_\tau^0$	$Y_t^0 = Y_b^0 = Y_\tau^0$
m_b		$m_0, m_{1/2}$	$m_0, m_{1/2}$
m_τ		$\tan \beta$	$\tan \beta$
M_Z		μ	μ
$b \rightarrow s\gamma$		(A_0)	A_0
$\tau_{universe}$			

Table 1: Summary of fit input and output variables.

The five-dimensional parameter space of the MSSM is big enough to be represented in a two- or three-dimensional picture. To make our analysis more clear, we consider various projections of the parameter space.

We first choose the value of the Higgs mixing parameter μ from the requirement of radiative EW symmetry breaking, then we take the values of $\tan \beta$ from the requirement of Yukawa coupling unification (see Fig.25). One finds two possible solutions: low $\tan \beta$ solution corresponding to $\tan \beta \approx 1.7$ and high $\tan \beta$ solution corresponding to $\tan \beta \approx 30 \div 60$. In what follows, we refer to these two solutions as low and high $\tan \beta$ scenarios, respectively.

What is left are the values of the soft parameters A , m_0 and $m_{1/2}$. However, the role of the trilinear coupling A is not essential since at low energies it runs to the infra-red fixed

point and is almost independent of initial conditions. Therefore, imposing the above-mentioned constraints, the parameter space of the MSSM is reduced to a two dimensional one. In what follows, we consider the plane $m_0, m_{1/2}$ and find the allowed region in this plane. Each point at this plane corresponds to a fixed set of parameters and allows one to calculate the spectrum, the cross-sections and other quantities of interest.

We present the allowed regions of the parameter space for low and high $\tan\beta$ scenarios in Fig.26. This plot demonstrates the role of various constraints in the χ^2 function. The contours enclose domains by the particular constraints used in the analysis [63]. In case when

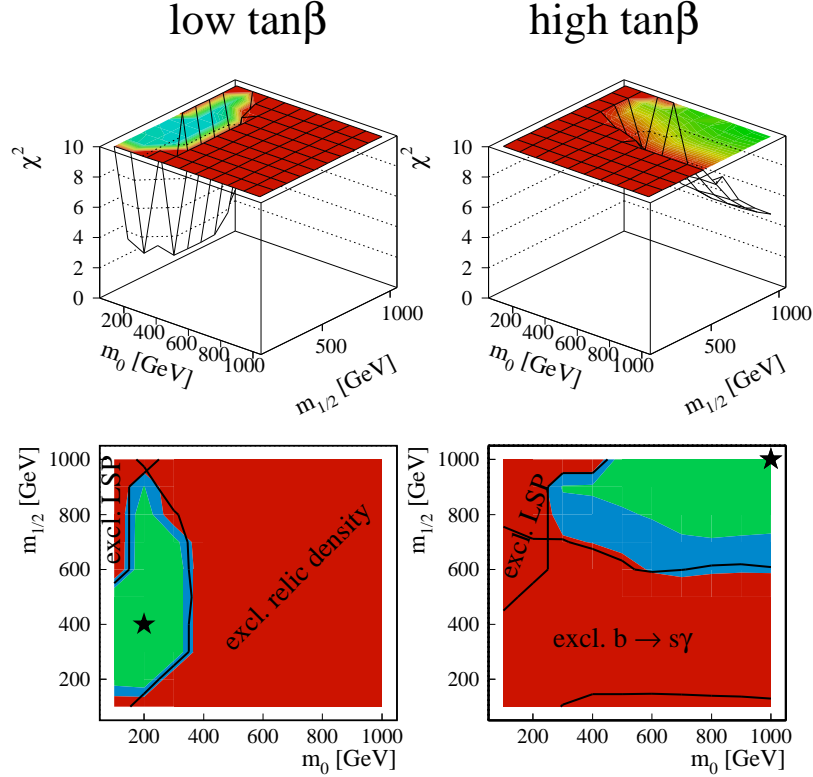


Figure 26: The χ^2 -distribution for low and high $\tan\beta$ solutions. The different shades in the projections indicate steps of $\Delta\chi^2 = 4$, so basically only the light shaded region is allowed. The stars indicate the optimum solution. Contours enclose domains by the particular constraints used in the analysis.

the requirement of the $b \rightarrow s\gamma$ decay rate is not taken into account (due to uncertainties of the high order contributions), the allowed region of parameter space becomes much wider, as it is illustrated in Fig.27. Now much lower values of m_0 and $m_{1/2}$ are allowed which lead to lower values of sparticle masses.

6.3 The mass spectrum of superpartners

When the parameter set is fixed, one can calculate the mass spectrum of superpartners. Below we show the set of parameters and the predicted mass spectrum corresponding to the best fit values indicated by stars in Fig.26 [39].

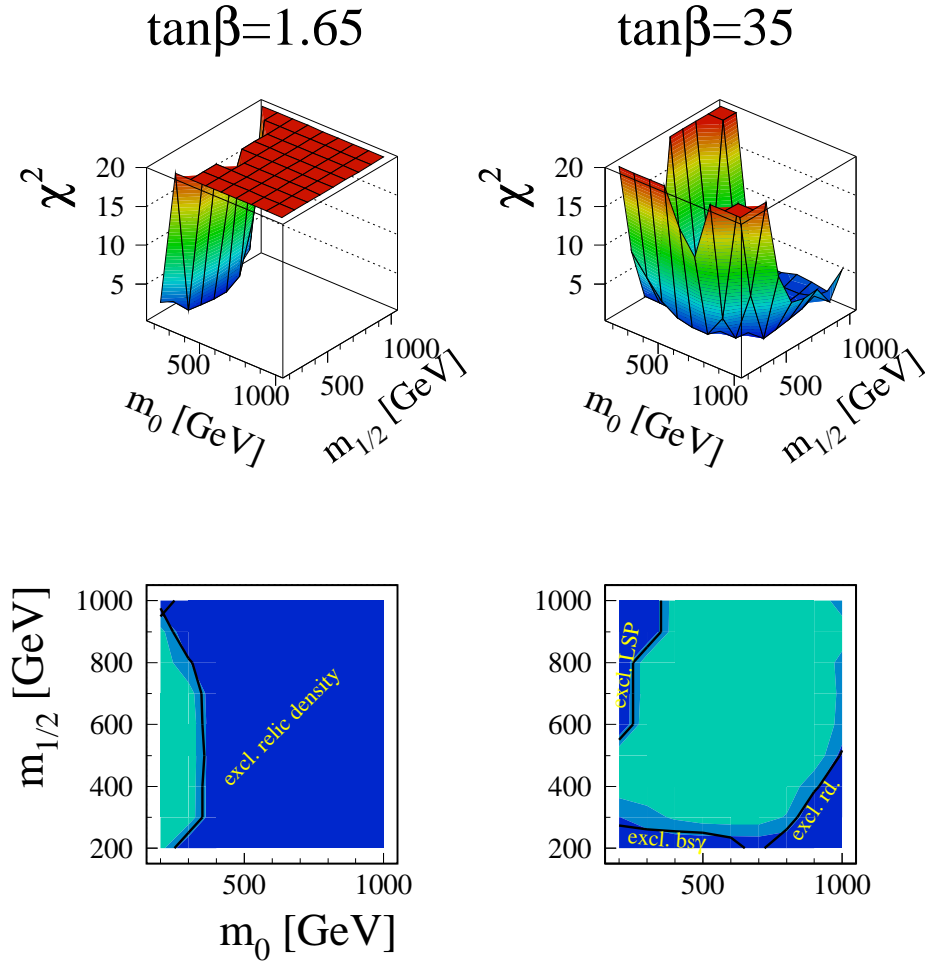


Figure 27: The same as Fig.26 but with the $b \rightarrow s\gamma$ constraint released with account taken of the higher order corrections [64].

Fitted SUSY parameters		
Symbol	low $\tan \beta$	high $\tan \beta$
$\tan \beta$	1.71	35.0
m_0	200	600
$m_{1/2}$	500	400
$\mu(0)$	1084	-558
$A(0)$	0	0
$1/\alpha_{GUT}$	24.8	24.8
M_{GUT}	$1.6 \cdot 10^{16}$	$1.6 \cdot 10^{16}$

Table 2: Values of the fitted SUSY parameters for low and high $\tan \beta$ (in GeV, when applicable).

To demonstrate the dependence of masses of the lightest particles on the choice of parameters, we show below in Figs.28,29 their values in the whole $m_0, m_{1/2}$ plane for the case of low and high $\tan \beta$ solutions, respectively [63]. One can see that the masses of gauginos (charginos and neutralinos) and Higgses basically depend on $m_{1/2}$, while those of squarks and sleptons on m_0 .

SUSY masses in [GeV]		
Symbol	low $\tan \beta$	high $\tan \beta$
$\tilde{\chi}_1^0(\tilde{B}), \tilde{\chi}_2^0(\tilde{W}^3)$	214, 413	170, 322
$\tilde{\chi}_3^0(\tilde{H}_1), \tilde{\chi}_4^0(\tilde{H}_2)$	1028, 1016	481, 498
$\tilde{\chi}_1^\pm(\tilde{W}^\pm), \tilde{\chi}_2^\pm(\tilde{H}^\pm)$	413, 1026	322, 499
\tilde{g}	1155	950
\tilde{e}_L, \tilde{e}_R	303, 270	663, 621
$\tilde{\nu}_L$	290	658
\tilde{q}_L, \tilde{q}_R	1028, 936	1040, 1010
$\tilde{\tau}_1, \tilde{\tau}_2$	279, 403	537, 634
\tilde{b}_1, \tilde{b}_2	953, 1010	835, 915
\tilde{t}_1, \tilde{t}_2	727, 1017	735, 906
h, H	95, 1344	119, 565
A, H^\pm	1340, 1344	565, 571

Table 3: Values of the SUSY mass spectra for the low and high $\tan \beta$ solutions given in Table 2.

$$\tan \beta = 1.65, \mu > 0$$

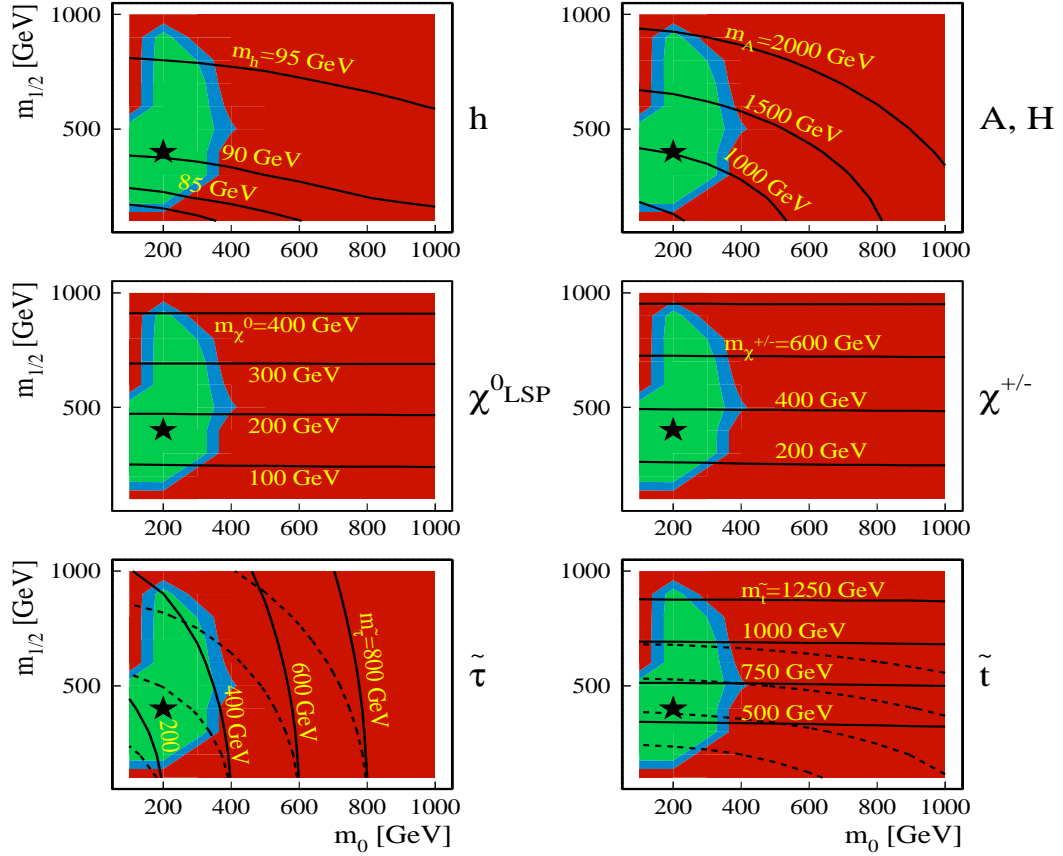


Figure 28: The masses of the lightest particles in the CMSSM for the low $\tan \beta$ scenario. The contours show the fixed mass values of the corresponding particles.

$\tan\beta=35$

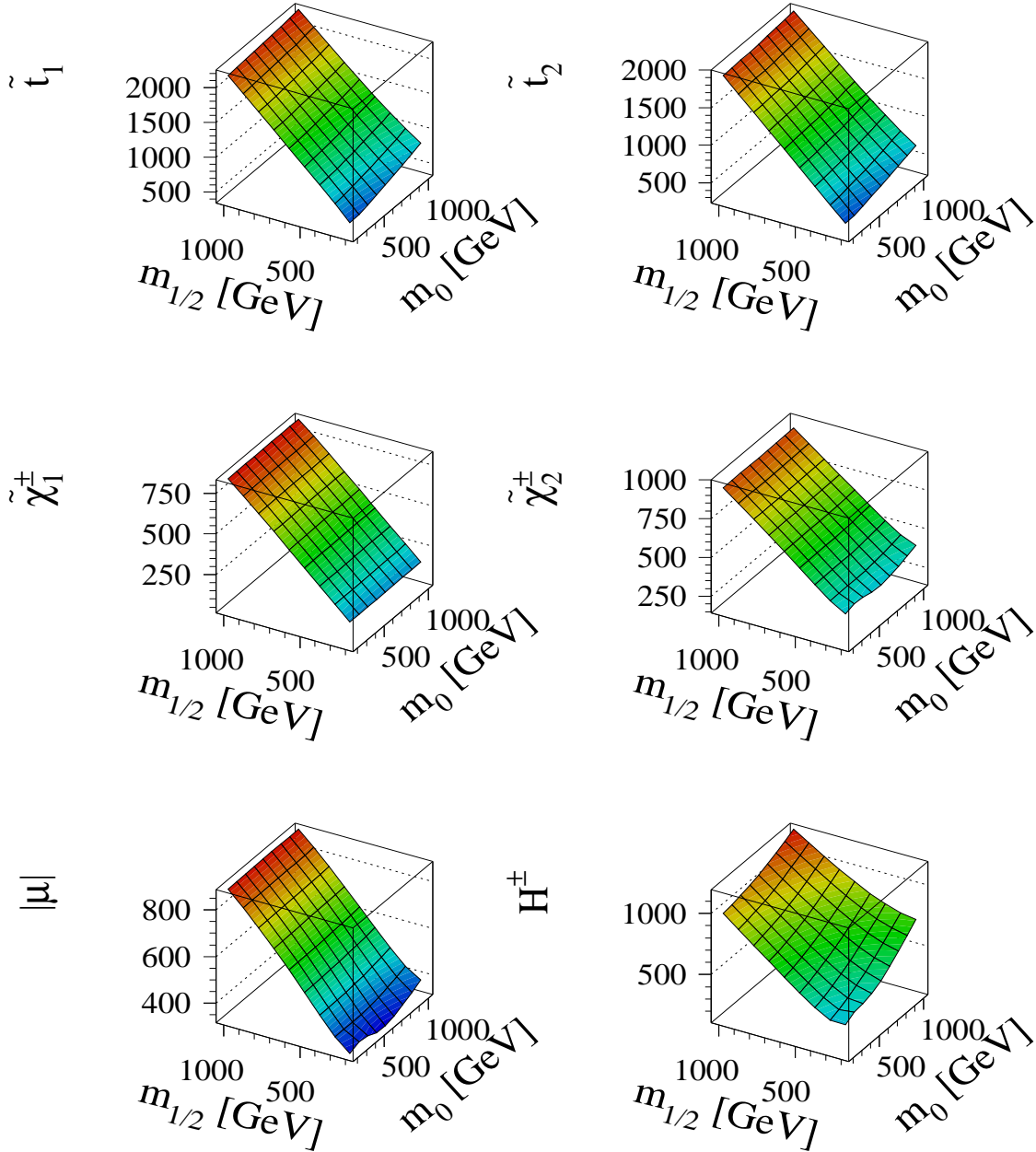


Figure 29: The masses of the particles and the Higgs mixing parameter μ for $\tan\beta = 35, \mu < 0$.

6.4 Experimental signatures at e^+e^- colliders

Experiments are finally beginning to push into a significant region of supersymmetry parameter space. We know the particles and their couplings, but we do not know their masses and mixings. Given the mass spectrum one can calculate the cross-sections and consider the possibilities of observing new particles at modern accelerators. Otherwise, one can get restrictions on unknown parameters.

We start with e^+e^- colliders and, first of all, with LEP II. In the leading order creation of superpartners is given by the diagrams shown in Fig.16 above. For a given center of mass energy the cross-sections depend on the mass of created particles and vanish at the kinematic boundary. For a sample example of c.m. energy of LEP II equal to 183 GeV, they are shown at Fig.30.

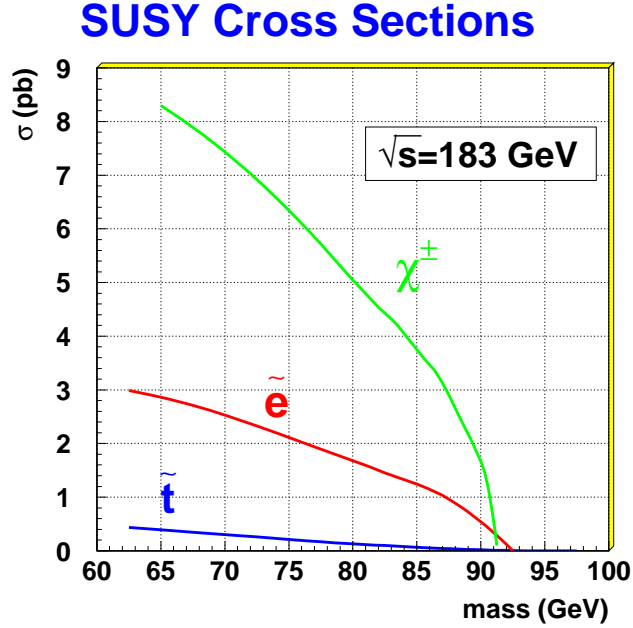


Figure 30: The cross-section of sparticle production at LEP II as functions of sparticle masses

Experimental signatures are defined by the decay modes which vary with the mass spectrum. The main ones are summarized below.

<u>Production</u>	<u>Key Decay Modes</u>	<u>Signatures</u>
• $\tilde{l}_{L,R}\tilde{l}_{L,R}$	$\tilde{l}_R^\pm \rightarrow l^\pm \tilde{\chi}_i^0 \searrow$ cascade $\tilde{l}_L^\pm \rightarrow l^\pm \tilde{\chi}_i^0 \nearrow$ decays	acomplanar pair of charged leptons + \cancel{E}_T
• $\tilde{\nu}\tilde{\nu}$	$\tilde{\nu} \rightarrow l^\pm \tilde{\chi}_1^0$	\cancel{E}_T
• $\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm$	$\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 l^\pm \nu$, $\tilde{\chi}_1^0 q \bar{q}'$ $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_2^0 f \bar{f}'$ $\tilde{\chi}_1^\pm \rightarrow l \tilde{\nu}_l \rightarrow l \nu_l \tilde{\chi}_1^0$ $\tilde{\chi}_1^\pm \rightarrow \nu_l \tilde{l} \rightarrow \nu_l l \tilde{\chi}_1^0$	isolated lepton + 2 jets + \cancel{E}_T pair of acomplanar leptons + \cancel{E}_T 4 jets + \cancel{E}_T

- $\tilde{\chi}_i^0 \tilde{\chi}_j^0 \quad \tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 X, \tilde{\chi}_j^0 \rightarrow \tilde{\chi}_1^0 X' \quad X = \nu_l \bar{\nu}_l \text{ invisible}$
 $= \gamma, 2l, 2 \text{ jets}$
 $2l + \cancel{E}_T, l + 2j + \cancel{E}_T$
- $\tilde{t}_i \tilde{t}_j \quad \tilde{t}_1 \rightarrow c \tilde{\chi}_1^0$
 $\tilde{t}_1 \rightarrow b \tilde{\chi}_1^\pm \rightarrow b f \bar{f}' \tilde{\chi}_1^0$
 $2 \text{ jets} + \cancel{E}_T$
 $2 \text{ b jets} + 2 \text{ leptons} + \cancel{E}_T$
 $2 \text{ b jets} + 2 \text{ jets} + \text{lepton} + \cancel{E}_T$
- $\tilde{b}_i \tilde{b}_j \quad \tilde{b}_i \rightarrow b \tilde{\chi}_1^0$
 $\tilde{b}_i \rightarrow b \tilde{\chi}_2^0 \rightarrow b f \bar{f}' \tilde{\chi}_1^0$
 $2 \text{ b jets} + \cancel{E}_T$
 $2 \text{ b jets} + 2 \text{ leptons} + \cancel{E}_T$
 $2 \text{ b jets} + 2 \text{ jets} + \cancel{E}_T$

A characteristic feature of all possible signatures is the missing energy and transverse momenta, which is a trade mark of a new physics.

Numerous attempts to find superpartners at LEP II gave no positive result thus imposing the lower bounds on their masses [54]. They are shown on the parameter plane in Figs.31,32.

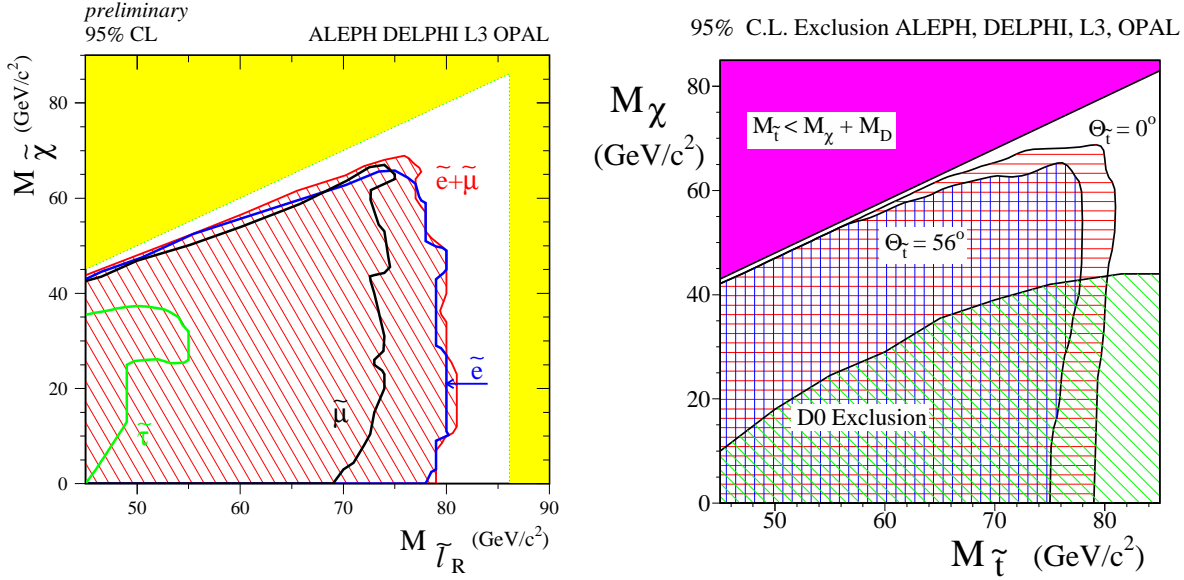


Figure 31: The excluded region in chargino-slepton and chargino-stop mass plane

In the case of stop masses, the result depends on the stop mixing angle $\Theta_{\tilde{t}}$ calculated from the stop mixing matrix. It defines the mass eigenstates basis \tilde{t}_1 and \tilde{t}_2

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos \Theta_{\tilde{t}} & \sin \Theta_{\tilde{t}} \\ -\sin \Theta_{\tilde{t}} & \cos \Theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}.$$

Nonobservation of charginos at the maximal LEP II energy defines the lower limit on chargino masses as shown in Fig.32 [54].

Typical LEP II limits on the masses of superpartners are

$$\begin{aligned} m_{\chi_1^0} &> 40 \text{ GeV} & m_{\tilde{e}_{L,R}} &> 105 \text{ GeV} & m_{\tilde{t}} &> 90 \text{ GeV} \\ m_{\chi_1^\pm} &> 100 \text{ GeV} & m_{\tilde{\mu}_{L,R}} &> 100 \text{ GeV} & m_{\tilde{b}} &> 80 \text{ GeV} \\ & & m_{\tilde{\tau}_{L,R}} &> 80 \text{ GeV} & & \end{aligned} \quad (6.5)$$

Preliminary DELPHI LSP limit at 189 GeV

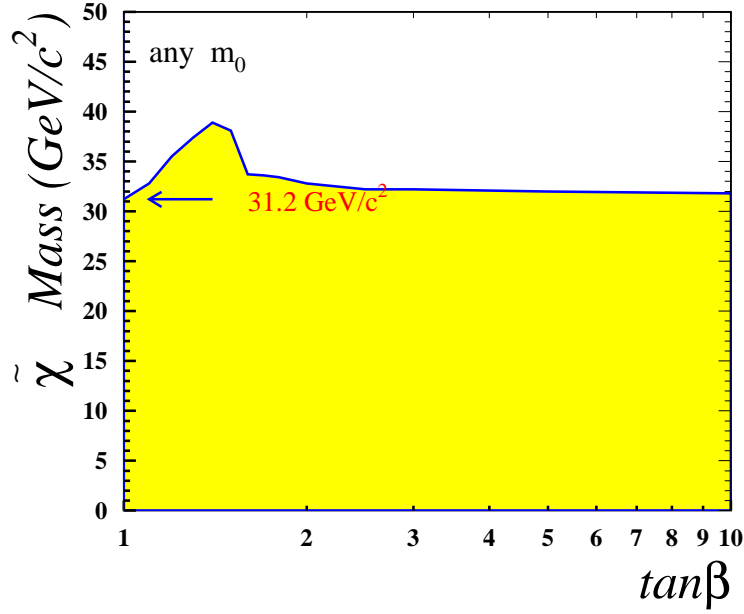


Figure 32: Cross section of chargino production at LEP and experimental limits on chargino mass

6.5 Experimental signatures at hadron colliders

Experimental signatures at hadron colliders are similar to those at e^+e^- machines; however, here one has much wider possibilities. Besides the usual annihilation channel identical to e^+e^- one with the obvious replacement of electrons by quarks (see Fig.33), one has numerous processes of gluon fusion, quark-antiquark and quark-gluon scattering (see Fig.34).

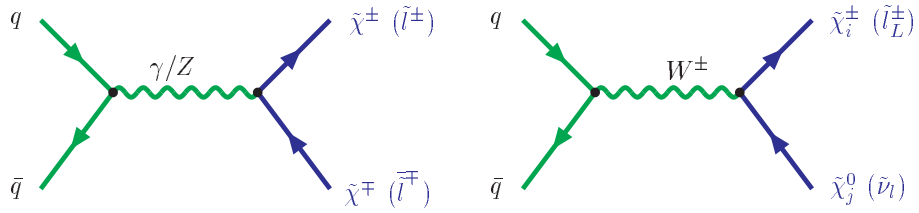


Figure 33: Annihilation channel

The final states depend on gluino decay modes. If squarks are heavier, i.e. $m_{\tilde{q}} > m_{\tilde{g}}$, then the main gluino decay modes are

$$\tilde{g} \rightarrow t + \bar{t} + \tilde{\chi}_i^0, \quad \tilde{g} \rightarrow t + \bar{b} + \tilde{\chi}_i^-, \quad \tilde{g} \rightarrow t + b + \tilde{\chi}_i^+,$$

otherwise gluino can decay into quarks and squarks with further decay of the latter.

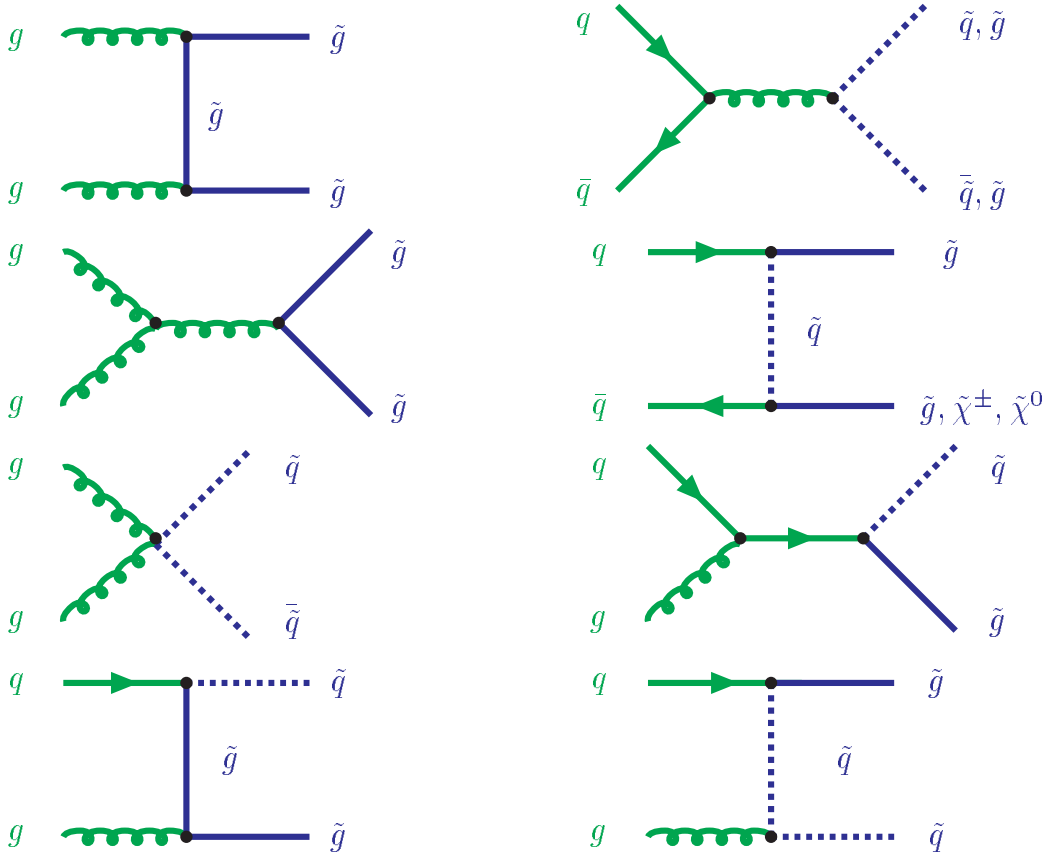


Figure 34: Gluon fusion, $q\bar{q}$ scattering, quark-gluon scattering

Experimental SUSY signatures at the Tevatron (and LHC) are

<u>Production</u>	<u>Key Decay Modes</u>	<u>Signatures</u>
<ul style="list-style-type: none"> $\tilde{g}\tilde{g}, \tilde{q}\tilde{q}, \tilde{g}\tilde{q}$ 	$\left. \begin{array}{l} \tilde{g} \rightarrow q\bar{q}\tilde{\chi}_1^0 \\ q\bar{q}'\tilde{\chi}_1^\pm \\ g\tilde{\chi}_1^0 \\ \tilde{q} \rightarrow q\tilde{\chi}_i^0 \\ \tilde{q} \rightarrow q'\tilde{\chi}_i^\pm \end{array} \right\} \begin{array}{l} m_{\tilde{q}} > m_{\tilde{g}} \\ m_{\tilde{g}} > m_{\tilde{q}} \end{array}$	\cancel{E}_T + multijets (+leptons)

• $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$	$\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 l^\pm \nu, \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 ll$ $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 q \bar{q}', \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 ll,$	Trilepton + \cancel{E}_T Dilepton + jet + \cancel{E}_T
• $\tilde{\chi}_1^+ \tilde{\chi}_1^-$	$\tilde{\chi}_1^+ \rightarrow l \tilde{\chi}_1^0 l^\pm \nu$	Dilepton + \cancel{E}_T
• $\tilde{\chi}_i^0 \tilde{\chi}_i^0$	$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 X, \tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 X'$	\cancel{E}_T + Dilepton + (jets) + (leptons)
• $\tilde{t}_1 \tilde{t}_1$	$\tilde{t}_1 \rightarrow c \tilde{\chi}_1^0$ $\tilde{t}_1 \rightarrow b \tilde{\chi}_1^\pm, \tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 l^\pm \nu, \tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 q \bar{q}'$ $\tilde{t}_1 \rightarrow b \tilde{\chi}_1^\pm, \tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 l^\pm \nu, \tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 l^\pm \nu$	2 acollinear jets + \cancel{E}_T single lepton + \cancel{E}_T + $b's$ Dilepton + \cancel{E}_T + $b's$
• $\tilde{l}, \tilde{l}\nu, \tilde{n}u\tilde{\nu}$	$\tilde{l}^\pm \rightarrow l \pm \tilde{\chi}_i^0, \tilde{l}^\pm \rightarrow \nu_l \tilde{\chi}_i^\pm$ $\tilde{\nu} \rightarrow \nu \tilde{\chi}_1^0$	Dilepton + \cancel{E}_T Single lepton + \cancel{E}_T + (jets) \cancel{E}_T

Note again the characteristic missing energy and transverse momenta events.

Contrary to e^+e^- colliders, at hadron machines the background is extremely rich and essential.

6.6 The lightest superparticle

One of the crucial questions is the properties of the lightest superparticle. Different SUSY breaking scenarios lead to different experimental signatures and different LSP.

- Gravity mediation

In this case, the LSP is the lightest neutralino $\tilde{\chi}_1^0$, which is almost 90% photino for a low $\tan\beta$ solution and contains more higgsino admixture for high $\tan\beta$. The usual signature for LSP is missing energy; $\tilde{\chi}_1^0$ is stable and is the best candidate for the cold dark matter in the Universe. Typical processes, where the LSP is created, end up with jets + \cancel{E}_T , or leptons + \cancel{E}_T , or both jets + leptons + \cancel{E}_T .

- Gauge mediation

In this case the LSP is the gravitino \tilde{G} which also leads to missing energy. The actual question here is what the NLSP, the next lightest particle, is. There are two possibilities:

i) $\tilde{\chi}_1^0$ is the NLSP. Then the decay modes are

$$\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}, h \tilde{G}, Z \tilde{G}.$$

As a result, one has two hard photons + \cancel{E}_T , or jets + \cancel{E}_T .

ii) \tilde{l}_R is the NLSP. Then the decay mode is $\tilde{l}_R \rightarrow \tau \tilde{G}$ and the signature is a charged lepton and the missing energy.

- Anomaly mediation

In this case, one also has two possibilities:

i) $\tilde{\chi}_1^0$ is the LSP and wino-like. It is almost degenerate with the NLSP.

ii) $\tilde{\nu}_L$ is the LSP. Then it appears in the decay of chargino $\tilde{\chi}^\pm \rightarrow \tilde{\nu} l$ and the signature is the charged lepton and the missing energy.

- R-parity violation

In this case, the LSP is no longer stable and decays into the SM particles. It may be charged (or even colored) and may lead to rare decays like neutrinoless double β -decay, etc.

Experimental limits on the LSP mass follow from non-observation of the corresponding events. Modern low limit is around 40 GeV (see Fig.35).

Preliminary DELPHI LSP limit at 189 GeV

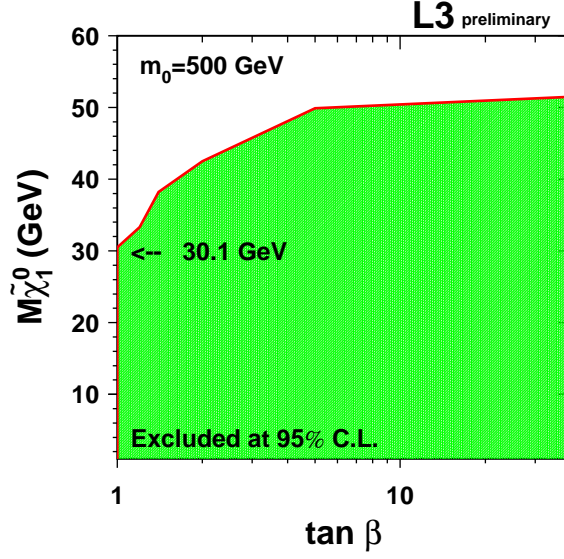
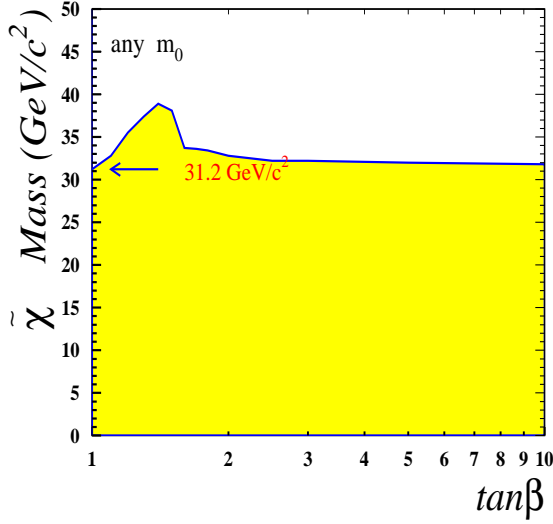


Figure 35: The LSP mass limits within the MSSM [54]

7 The Higgs boson in the SM and the MSSM

One of the hottest topics in the SM now is the search for the Higgs boson. It is also a window to a new physics. Below we consider the situation with the Higgs boson search and the properties of the Higgs boson in the MSSM.

7.1 Allowed mass range in the SM

The last unobserved particle from the Standard Model is the Higgs boson [65]. Its discovery would allow one to complete the SM paradigm and confirm the mechanism of spontaneous symmetry breaking. On the contrary, the absence of the Higgs boson would awake doubts about the whole picture and would require new concepts.

Experimental limits on the Higgs boson mass come from a direct search at LEP II and Tevatron and from indirect fits of electroweak precision data, first of all from the radiative corrections to the W and top quark masses. A combined fit of modern experimental data gives [66]

$$m_h = 90^{+55}_{-47} \text{ GeV}, \quad (7.1)$$

which at the 95% confidence level leads to the upper bound of 200 GeV (see Fig.36). At the same time, recent direct searches at LEP II for the c.m. energy of 209 GeV give the lower limit of 113.4 GeV[66]. From a theoretical point of view a low Higgs mass could be a hint for physics beyond the SM, in particular, for the supersymmetric extension of the SM.

Within the Standard Model the value of the Higgs mass m_h is not predicted. However, one can get the bounds on the Higgs mass [40, 67]. They follow from the behaviour of the quartic coupling which is related to the Higgs mass by eqs.(1.9,1.13) $m_h^2 = 2\lambda v$ and obeys the following renormalization group equation describing the change of λ with a scale:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2} (6\lambda^2 + 6\lambda y_t^2 - 6y_t^4 + \text{gauge terms}) \quad (7.2)$$

with $t = \ln(Q^2/\mu^2)$. Here y_t is the top-quark Yukawa coupling.

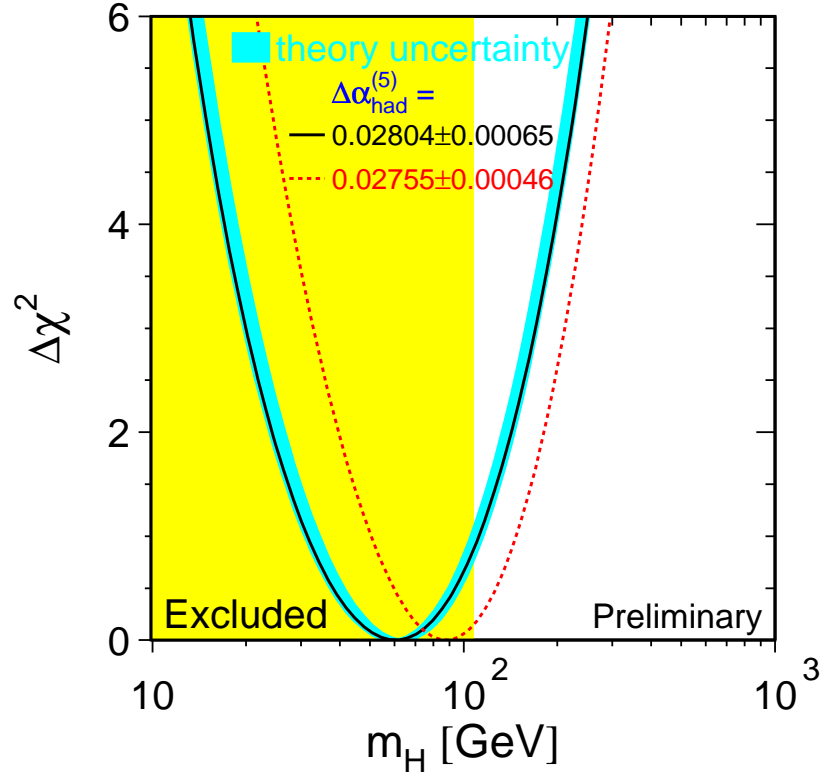


Figure 36: The χ^2 distribution as a function of the Higgs mass from the SM fit to the electroweak precision observables and the top mass. The shaded area is excluded by the direct searches.

Since the quartic coupling grows with rising energy infinitely and reaches the Landau pole, the upper bound on m_h follows from the requirement that the theory be valid up to the scale M_{Planck} or up to a given cut-off scale Λ below M_{Planck} [40]. The scale Λ could be identified with the scale at which the Landau pole develops. The upper bound on m_h depends mildly on the top-quark mass through the impact of the top-quark Yukawa coupling on the running of the quartic coupling λ in eq.(7.2).

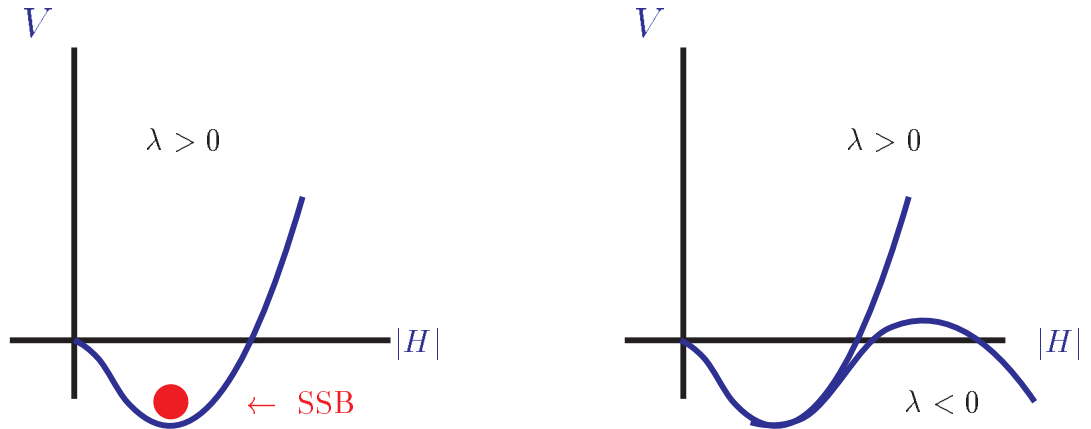


Figure 37: The shape of the Higgs potential

On the other hand, the requirement of vacuum stability in the SM (positivity of λ) imposes a lower bound on the Higgs boson mass, which crucially depends on both the top-quark mass and the cut-off Λ [40, 67]. Again, the dependence of this lower bound on m_t is due to the effect of the top-quark Yukawa coupling on the quartic coupling in eq.(7.2), which drives λ to negative values at large scales, thus destabilizing the standard electroweak vacuum (see Figs.37).

From the point of view of LEP and Tevatron physics, the upper bound on the SM Higgs boson mass does not pose any relevant restriction. The lower bound on m_h , instead, is particularly important in view of the search for a Higgs boson at LEP II and Tevatron. For $m_t \sim 174$ GeV and $\alpha_s(M_Z) = 0.118$ the running of the Higgs quartic coupling is shown in Fig.38. The results

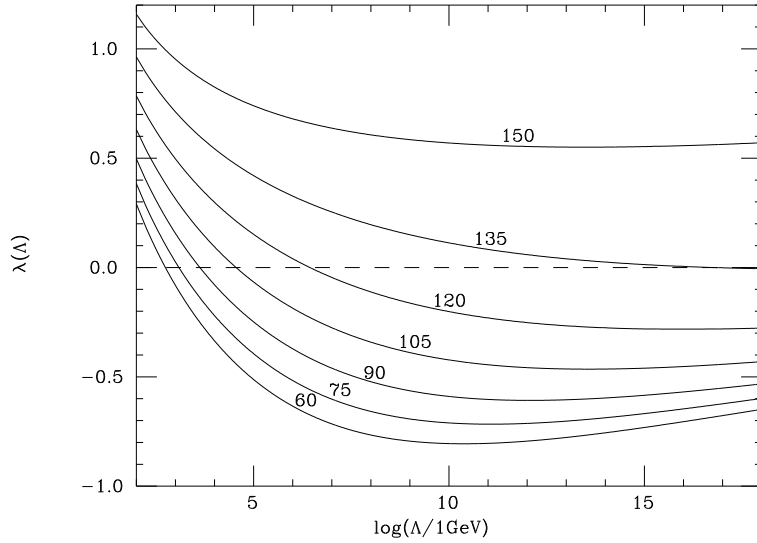


Figure 38: The running of the Higgs quartic coupling. Numbers shown above the lines indicate the value of the Higgs mass in GeV.

at $\Lambda = 10^{19}$ GeV or at $\Lambda = 1$ TeV can be given by the approximate formulae [67]

$$m_h > 135 + 2.1[m_t - 174] - 4.5 \left[\frac{\alpha_s(M_Z) - 0.118}{0.006} \right], \quad \Lambda = 10^{19} \text{ GeV}, \quad (7.3)$$

$$m_h > 72 + 0.9[m_t - 174] - 1.0 \left[\frac{\alpha_s(M_Z) - 0.118}{0.006} \right], \quad \Lambda = 1 \text{ TeV}, \quad (7.4)$$

where the masses are in units of GeV.

Fig.39 [68] shows the perturbativity and stability bounds on the Higgs boson mass of the SM for different values of the cut-off Λ at which new physics is expected. We see from Fig.39 and eqs.(7.3,7.4) that indeed for $m_t \sim 174$ GeV the discovery of a Higgs particle at LEP II would imply that the Standard Model breaks down at a scale Λ well below M_{GUT} or M_{Planck} , smaller for lighter Higgs. Actually, if the SM is valid up to $\Lambda \sim M_{GUT}$ or M_{Planck} , for $m_t \sim 174$ GeV only a small range of values is allowed: $134 < m_h < \sim 200$ GeV. For $m_t = 174$ GeV and $m_h < 100$ GeV [i.e. in the LEP II range] new physics should appear below the scale $\Lambda \sim$ a few to 100 TeV. The dependence on the top-quark mass however is noticeable. A lower value, $m_t \simeq 170$ GeV, would relax the previous requirement to $\Lambda \sim 10^3$ TeV, while a heavier value $m_t \simeq 180$ GeV would demand new physics at an energy scale as low as 10 TeV.

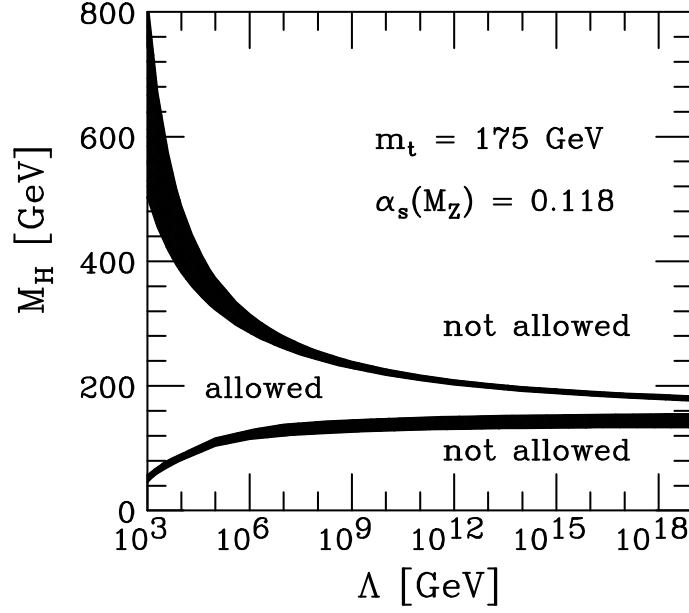


Figure 39: Strong interaction and stability bounds on the SM Higgs boson mass. Λ denotes the energy scale up to which the SM is valid.

7.2 SM Higgs production at LEP

The dominant mechanism for the Higgs boson production at LEP is the Higgsstrahlung. The Higgs boson is produced together with the Z^0 boson. A small contribution to the cross section comes also from the WW- and ZZ- fusion processes (see Fig.40). The cross section depends on

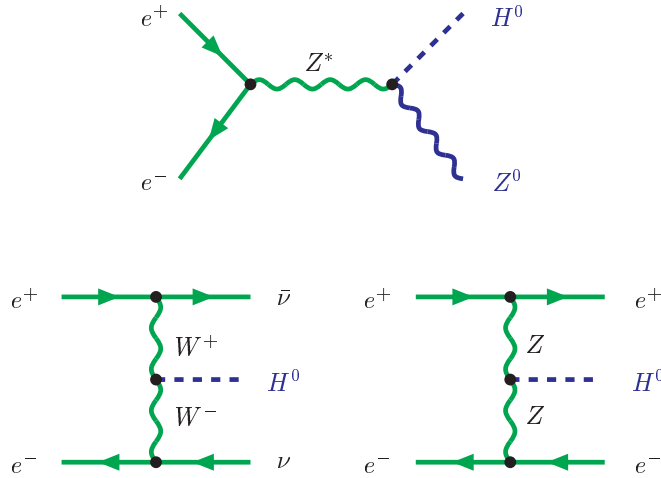


Figure 40: SM Higgs production at LEP: Higgsstrahlung (above) and WW- and ZZ- fusion (below)

the Higgs boson mass and decreases with increase of the latter. On the other hand, it grows with the centre of mass energy, as shown in Fig.41 [69]. Kinematic limit on the Higgs production is given by the c.m. energy minus the Z -boson mass.

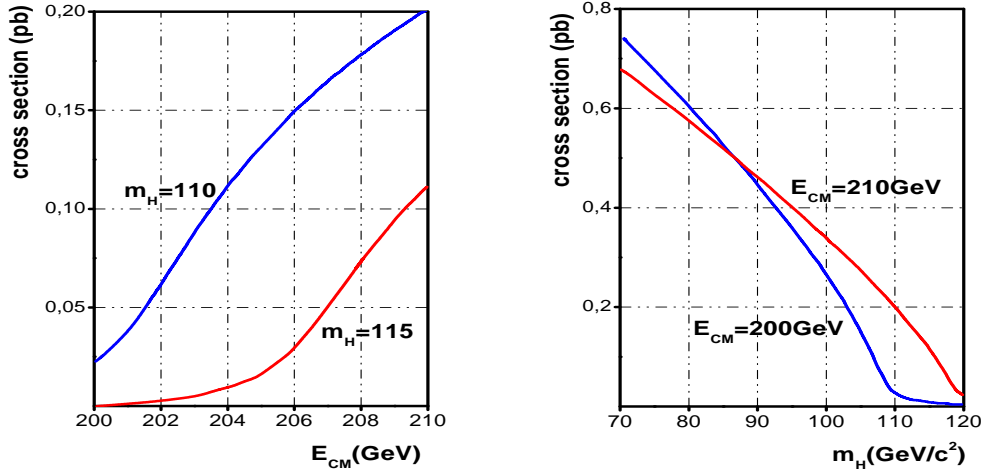


Figure 41: The cross section of the Higgs production at LEP II

However, one of the main problems is to distinguish the final products of the Higgs boson decay from the background, mainly the ZZ pair production. The branching ratios for the Higgs boson decay are shown in Fig.42. The Z boson has the same decay modes with different branchings. In the final states, one has either four hadronic jets, or two jets and two leptons, or

HZ final states (Higgsstrahlung production)

Channel	BR	Topology
$Hq\bar{q}$	64.6%	
$H\nu\bar{\nu}$	20.0%	
$He^+e^-, H\mu^+\mu^-$	6.7%	
$H\tau^+\tau^-, \tau^+\tau^-q\bar{q}$	8.7%	

Figure 42: The final states of the Higgs boson decay with the branching ratios

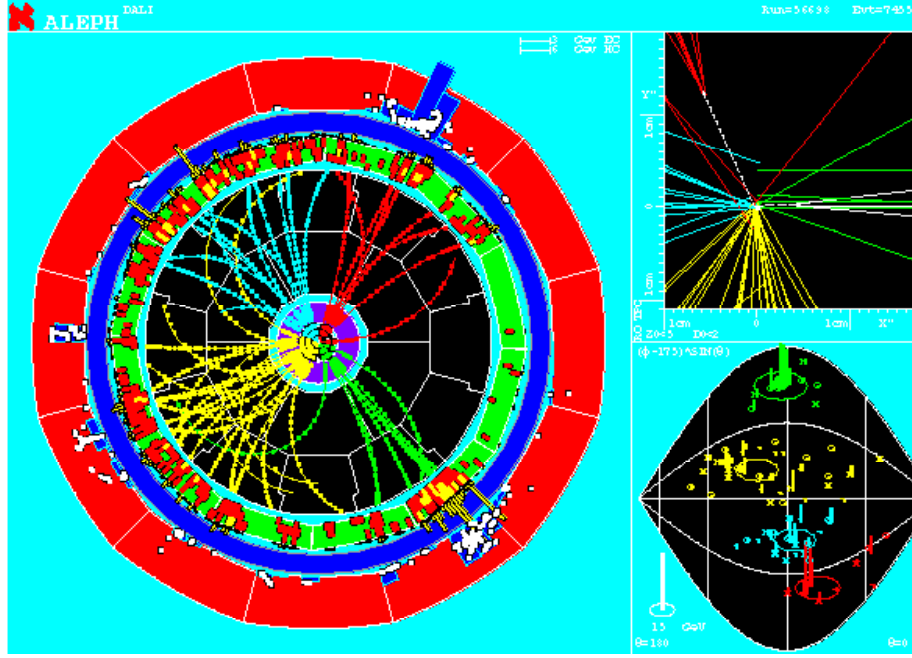


Figure 43: Typical four jet event

four leptons. The most probable is the four jet configuration, which is the most difficult from the point of view of unwanted background. A two-jet and two-lepton final state is more clean though less probable.

Attempts to find the Higgs boson have not met success so far. All the data are consistent with the background. An interesting four-jet event is shown in Fig.43 and is most likely a ZZ candidate [70]. A reconstructed invariant mass of two jets does not show noticeable deviation from background expectation. For the 68.1 background events expected, there are 70 events observed. The reconstructed Higgs mass for four-jet events is shown in Fig.44. At this kind of plots the real Higgs boson should give a peak above the background, as is shown for a would be Higgs mass of 110 GeV in Fig.44 [70].

Combined results from four LEP collaborations (ALEPH, DELPHI, L3 and OPAL) in the energy interval $\sqrt{s} = 200 - 210$ GeV allow one to get a lower limit on the Higgs mass. As it follows from Fig.45, at the 95% confidence level it is [66]

$$m_h > 113.3 \text{ GeV}/c^2 \quad @ 95\% \text{ C.L.} \quad (7.5)$$

Recent hot news from the LEP II accelerator show slight excess of events in hadronic channels. For the hard cuts keeping only "really good" events one can achieve the signal/background ratio of 2 with a few signal events indicating the 114 GeV Higgs boson (see Fig.46). Deviation from the background achieves 2.9 standard deviations and is better seen in the confidence level plots [70]. There are also some events in leptonic channel [71]. However, statistics is not enough to make definite conclusions.

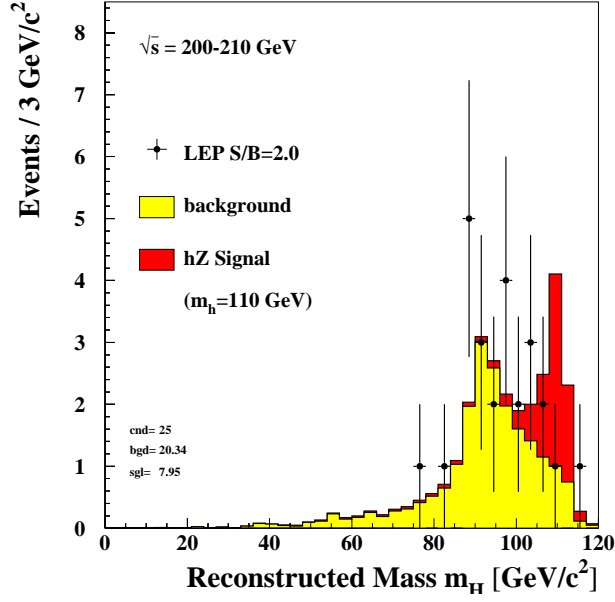


Figure 44: Reconstructed Higgs mass for four jet events. The peak shown in red corresponds to a would be Higgs boson with mass of 110 GeV

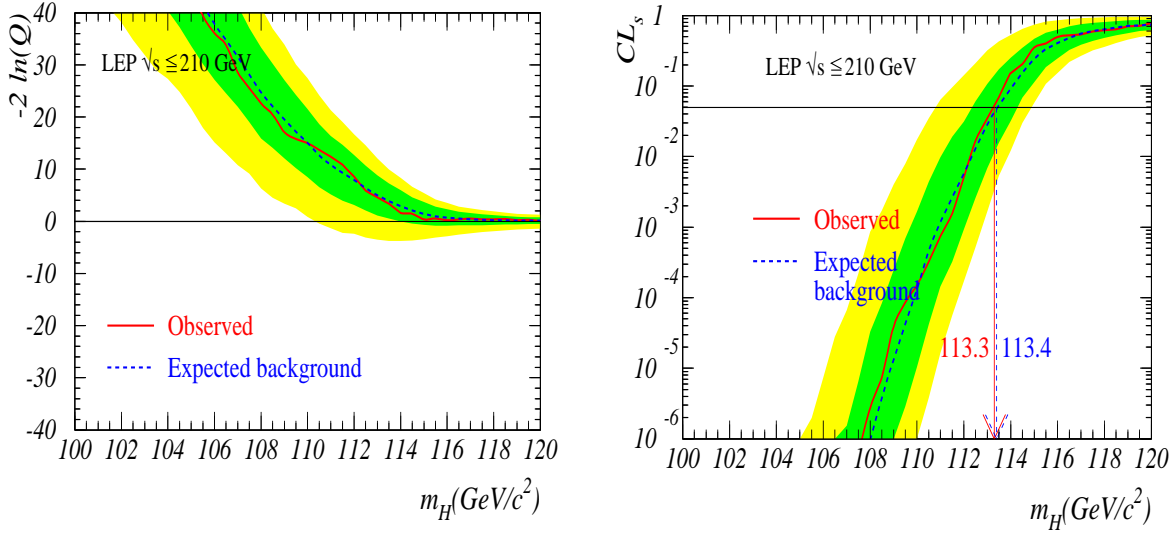


Figure 45: Combined Confidence Level plots for the Higgs searches at LEP

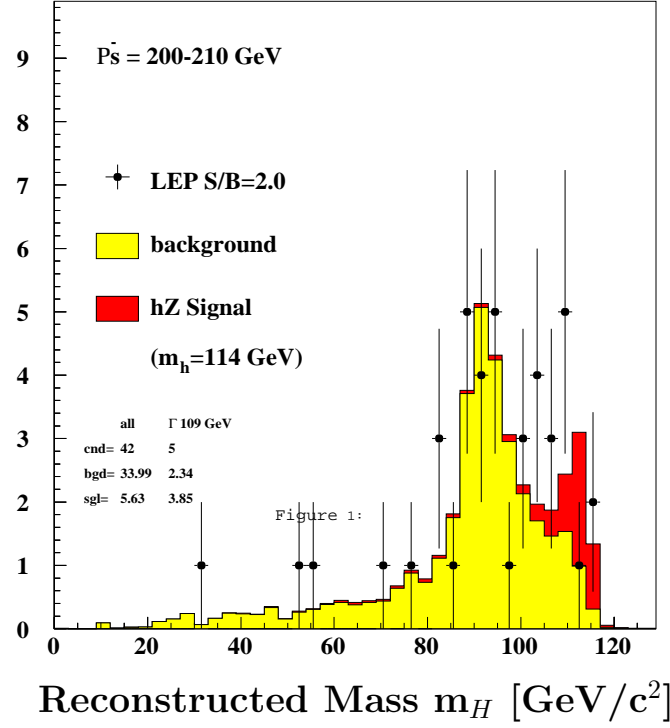
7.3 The Higgs boson mass in the MSSM

It has already been mentioned that in the MSSM the mass of the lightest Higgs boson is predicted to be less than the Z -boson mass. This is, however, the tree level result and the masses acquire the radiative corrections.

With account taken of the radiative corrections, the effective Higgs bosons potential is

$$V_{Higgs}^{eff} = V_{tree} + \Delta V, \quad (7.6)$$

Events/3 GeV/c²



	Data	Backg	Signal
All m_{rec}	42	34.0	5.6
$m_{rec} > 109 \text{ GeV}$	5	2.3	3.9

Figure 46: Higgs Candidate for 114 GeV/c²

where V_{tree} is given by eq.(5.16) and in the one-loop order

$$\Delta V_{1loop} = \sum_k \frac{1}{64\pi^2} (-1)^{J_k} (2J_k + 1) c_k m_k^4 \left(\log \frac{m_k^2}{Q^2} - \frac{3}{2} \right). \quad (7.7)$$

Here the sum is taken over all the particles in the loop, J_k is the spin and m_k is the field dependent mass of a particle at the scale Q .

The main contribution comes from the diagrams shown in Fig.47. These radiative corrections

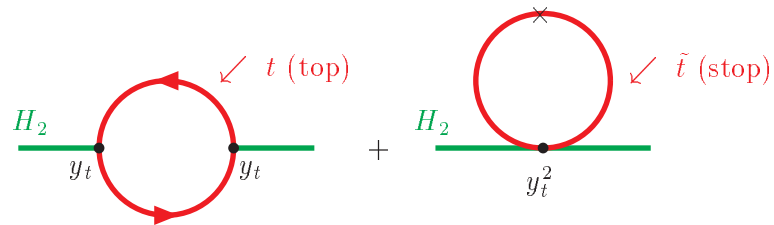


Figure 47: Corrections to the Higgs boson self-energy from the top(stop) loops

vanish when supersymmetry is not broken and are positive in the softly broken case. They are proportional to the mass squared of top (stop) quarks and depend on the values of the soft breaking parameters. Contributions from the other particles are much smaller [72, 73, 74]. The leading contribution comes from (s)top loops

$$\Delta V_{1loop}^{stop} = \frac{3}{32\pi^2} \left[\tilde{m}_{t_1}^4 \left(\log \frac{\tilde{m}_{t_1}^2}{Q^2} - \frac{3}{2} \right) + \tilde{m}_{t_2}^4 \left(\log \frac{\tilde{m}_{t_2}^2}{Q^2} - \frac{3}{2} \right) - 2m_t^4 \left(\log \frac{m_t^2}{Q^2} - \frac{3}{2} \right) \right]. \quad (7.8)$$

These corrections lead to the following modification of the tree-level relation for the lightest Higgs mass

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}_{t_1}^2 \tilde{m}_{t_2}^2}{m_t^4}. \quad (7.9)$$

One finds that the one-loop correction is positive and increases the mass value. Two loop corrections have the opposite effect but are smaller and result in slightly lower value of the Higgs mass [75, 63, 76].

To find out numerical values of these corrections, one has to determine the masses of all superpartners. Within the Constrained MSSM, imposing various constraints, one can define the

$$\tan \beta = 1.65, \mu < 0 \quad \tan \beta = 1.65, \mu > 0$$

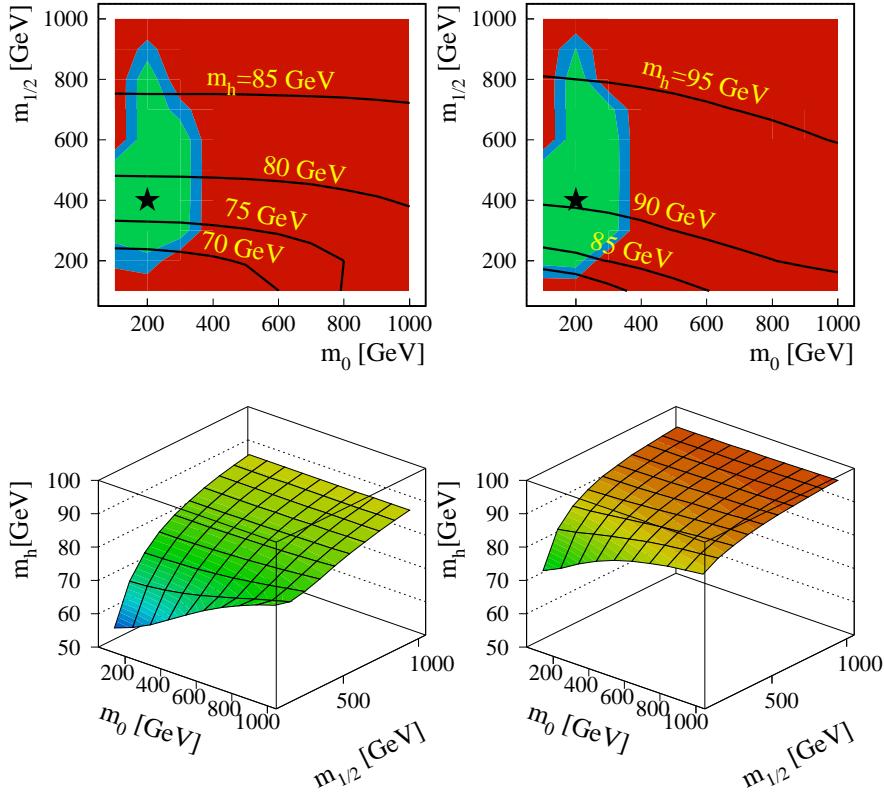


Figure 48: The mass of the lightest Higgs boson for the low $\tan \beta$ solution as a function of m_0 and $m_{1/2}$. The contours at the upper plots correspond to fixed values of the Higgs mass. The lower plots demonstrate the saturation of the mass at high values of mass parameters.

allowed region in the parameter space and calculate the spectrum of superpartners and, hence, the radiative corrections to the Higgs boson mass (see Figs.48, 49).

The Higgs mass depends mainly on the following parameters: the top mass, the squark masses, the mixing in the stop sector, the pseudoscalar Higgs mass and $\tan\beta$. As will be shown below, the maximum Higgs mass is obtained for large $\tan\beta$, for a maximum value of the top and squark masses and a minimum value of the stop mixing.

Note that in the CMSSM the Higgs mixing parameter μ is determined by the requirement of EWSB, which yields large values for μ [39]. Given that the pseudoscalar Higgs mass increases rapidly with μ , this mass is always much larger than the lightest Higgs mass and thus decouples. This decoupling is effective for all regions of the CMSSM parameter space, i.e. the lightest Higgs has the couplings of the SM Higgs within a few per cent. We present the value of the lightest

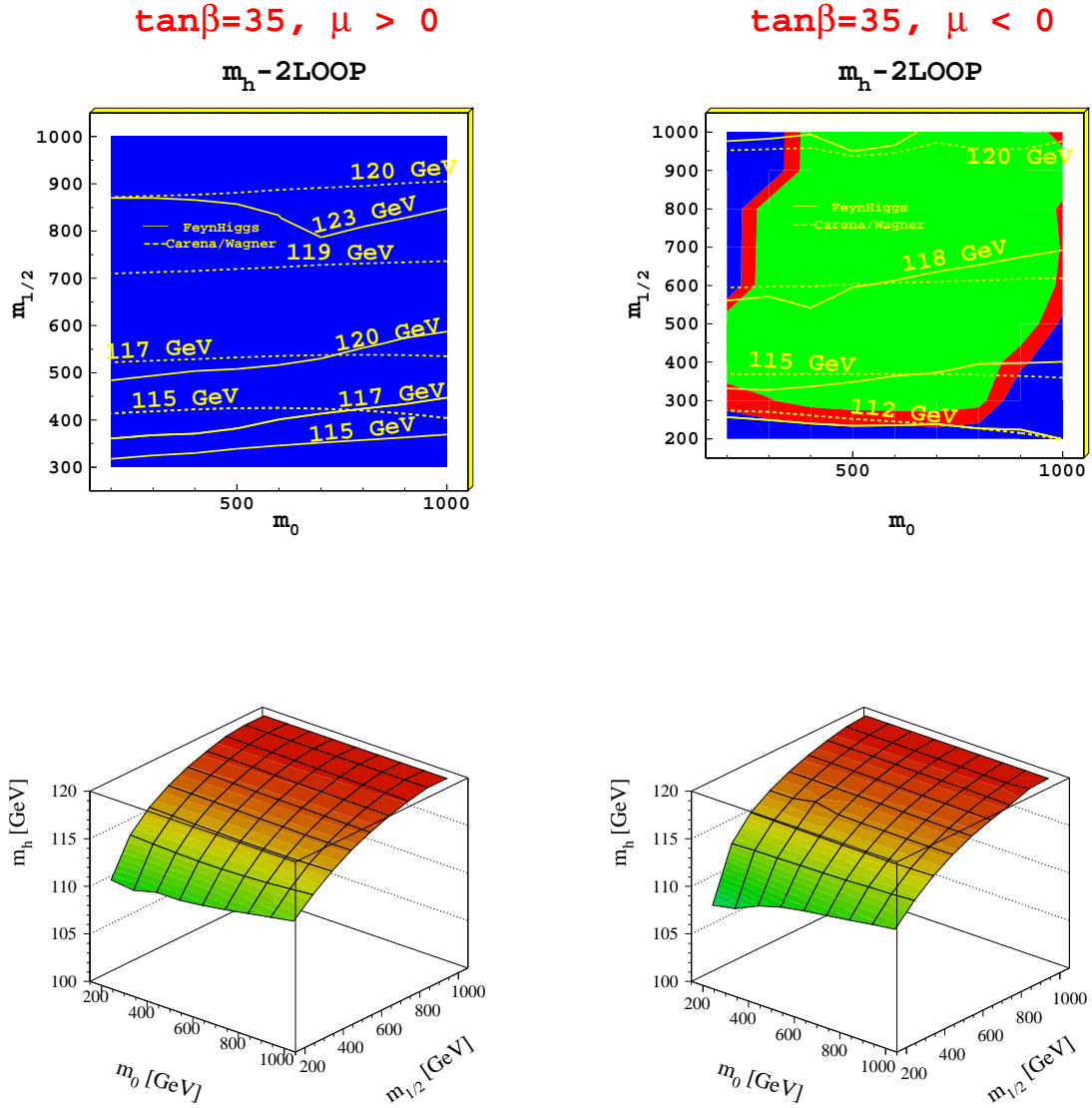


Figure 49: The same as in Fig. 48 but for the high $\tan\beta$ solution $\tan\beta = 35$.

Higgs mass in the whole $m_0, m_{1/2}$ plane for low and high $\tan\beta$ solutions, respectively [64] in Figs.48, 49. One can see that it is practically constant in the whole plane and is saturated for high values of m_0 and $m_{1/2}$.

The lightest Higgs boson mass m_h is shown as a function of $\tan\beta$ in Fig. 50 [64]. The shaded band corresponds to the uncertainty from the stop mass and stop mixing for $m_t = 175$ GeV. The upper and lower lines correspond to $m_t=170$ and 180 GeV, respectively.

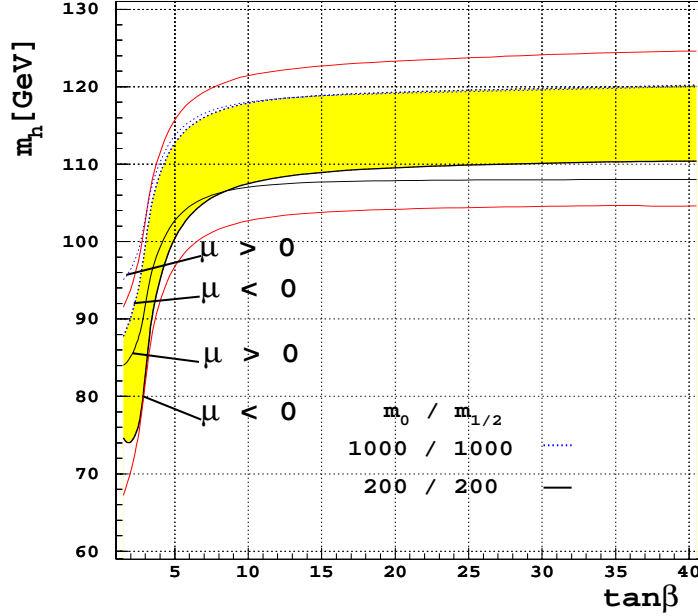


Figure 50: The mass of the lightest Higgs boson as a function of $\tan\beta$

The parameters used for the calculation of the upper limit are: $m_t = 180$ GeV, $A_0 = -3m_0$ and $m_0 = m_{1/2} = 1000$ GeV. The lowest line of the same figure gives the minimal values of m_h . For high $\tan\beta$ the values of m_h range from 105 GeV to 125 GeV. At present, there is no preference for any of the values in this range but it can be seen that the 95% C.L. lower limit on the Higgs mass [66] of 113.3 GeV excludes $\tan\beta < 3.3$.

In order to better understand the Higgs mass uncertainties, the relevant parameters were varied one by one. The largest uncertainty on the light Higgs mass originates from the stop masses. The Higgs mass varies between 110 and 120 GeV, if m_0 and $m_{1/2}$ are varied between 200 and 1000 GeV, which implies stop masses varying between 400 and 2000 GeV. Since at present there is no preference for any of the values between 110 and 120 GeV, the variance for a flat probability distribution is $10/\sqrt{12}=3$ GeV, which we take as an error estimate.

The remaining uncertainty of the Higgs mass originates from the mixing in the stop sector when one leaves A_0 as a free parameter. The mixing is determined by the off-diagonal element in the stop mass matrix $X_t = A_t - \mu/\tan\beta$. Its influence on the Higgs mass is quite small in the CMSSM since the low energy value A_t tends to a fixed point so that the stop mixing parameter $X_t = A_t - \mu/\tan\beta$ is not strongly dependent on A_0 . Furthermore, the μ term is not important at large $\tan\beta$. If we vary A_0 between $\pm 3m_0$, the error from the stop mixing in the Higgs boson mass is estimated to be ± 1.5 GeV. The values of $m_0 = m_{1/2} = 370$ GeV yield the central value of $m_h = 115$ GeV.

Given the uncertainty on the top mass of 5.2 GeV [43] leads to the uncertainty for the Higgs mass at large $\tan\beta$ of ± 5 GeV.

The uncertainties from the higher order calculations (HO) is estimated to be 2 GeV from a comparison of the full diagrammatic method [76] and the effective potential approach [75]. So combining all the uncertainties discussed before the results for the Higgs mass in the CMSSM can be summarized as follows:

- The low $\tan \beta$ scenario ($\tan \beta < 3.3$) of the CMSSM is excluded by the lower limit on the Higgs mass of 113.3 GeV [66].
- For the high $\tan \beta$ scenario the Higgs mass is found to be in the range from 110 to 120 GeV for $m_t = 175$ GeV. The central value is found to be [64]:

$$m_h = 115 \pm 3 \text{ (stopmass)} \pm 1.5 \text{ (stopmixing)} \pm 2 \text{ (theory)} \pm 5 \text{ (topmass)} \text{ GeV}, \quad (7.10)$$

where the errors are the estimated standard deviations around the central value. This prediction is independent of $\tan \beta$ for $\tan \beta > 20$ and decreases for lower $\tan \beta$.

However, these SUSY limits on the Higgs mass may not be so restricting if non-minimal SUSY models are considered. In a SUSY model extended by a singlet, the so-called Next-to-Minimal model, eq.(5.26) is modified and at the tree level the upper bound looks like [77]

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta, \quad (7.11)$$

where λ is an additional singlet Yukawa coupling. This coupling being unknown brings us back to the SM situation, though its influence is reduced by $\sin 2\beta$. As a result, for low $\tan \beta$ the upper bound on the Higgs mass is slightly modified (see Fig.51).

Even more dramatic changes are possible in models containing non-standard fields at intermediate scales. These fields appear in scenarios with gauge mediated supersymmetry breaking. In this case, the upper bound on the Higgs mass may increase up to 155 GeV [77] (the upper curve in Fig.51), though it is not necessarily saturated. One should notice, however, that these more sophisticated models do not change the generic feature of SUSY theories, the presence of the light

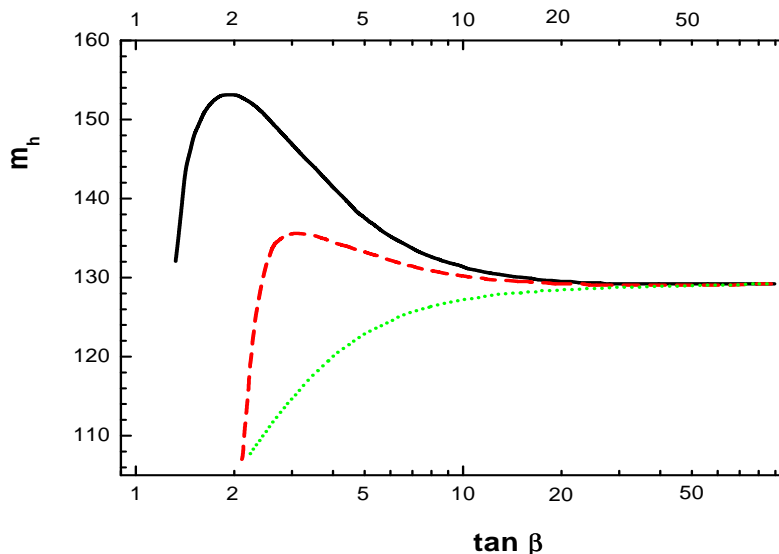


Figure 51: Dependence of the upper bound on the lightest Higgs boson mass on $\tan \beta$ in MSSM (lower curve), NMSSM (middle curve) and extended SSM (upper curve)

7.4 Perspectives of observation

LEP

In the case of supersymmetry, contrary to the SM, there are two competing processes for neutral Higgs production. Besides the usual Higgsstrahlung diagram there is also the pair production one when two Higgs bosons (the usual one and the pseudoscalar boson A) are produced. The cross-sections of these two processes are complimentary and related to the SM one by a simple formula (see Fig.52). Thus, the cross-section for Higgs production in the MSSM is usu-

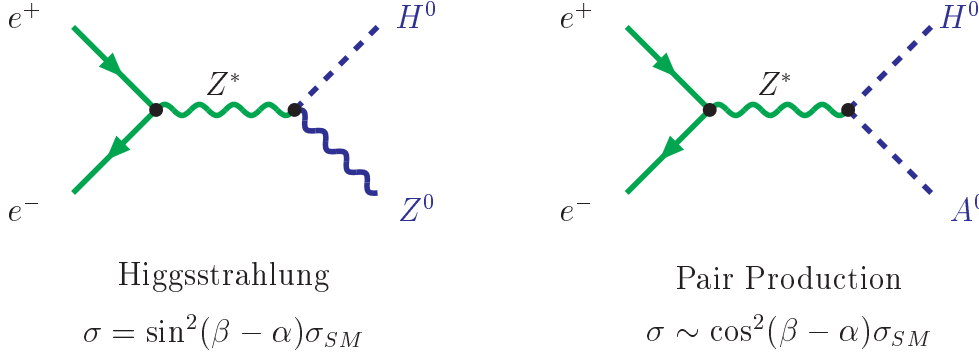


Figure 52: MSSM Higgs production at LEP: complimentary diagrams

ally lower than that of the SM. Therefore, searches for pair production are limited by a low cross-section rather than by a threshold (see Fig.53).

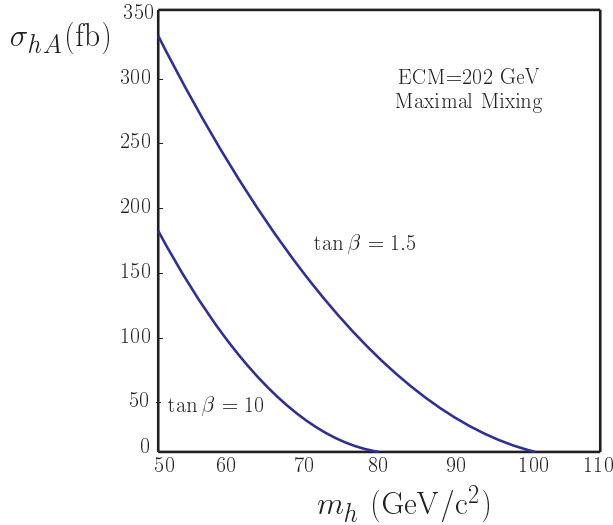


Figure 53: hA pair production cross section in fb as a function of m_h and $\tan \beta$

Non-observation of the Higgs boson at LEP in general gives lower bound on the Higgs boson mass than that in the SM. Modern experimental limits on the MSSM Higgs bosons are [69]

$$m_h > 90.5 \text{ GeV}/c^2, \quad m_A > 90.5 \text{ GeV}/c^2 \quad @ \quad 95\% \text{ C.L.} \quad (7.12)$$

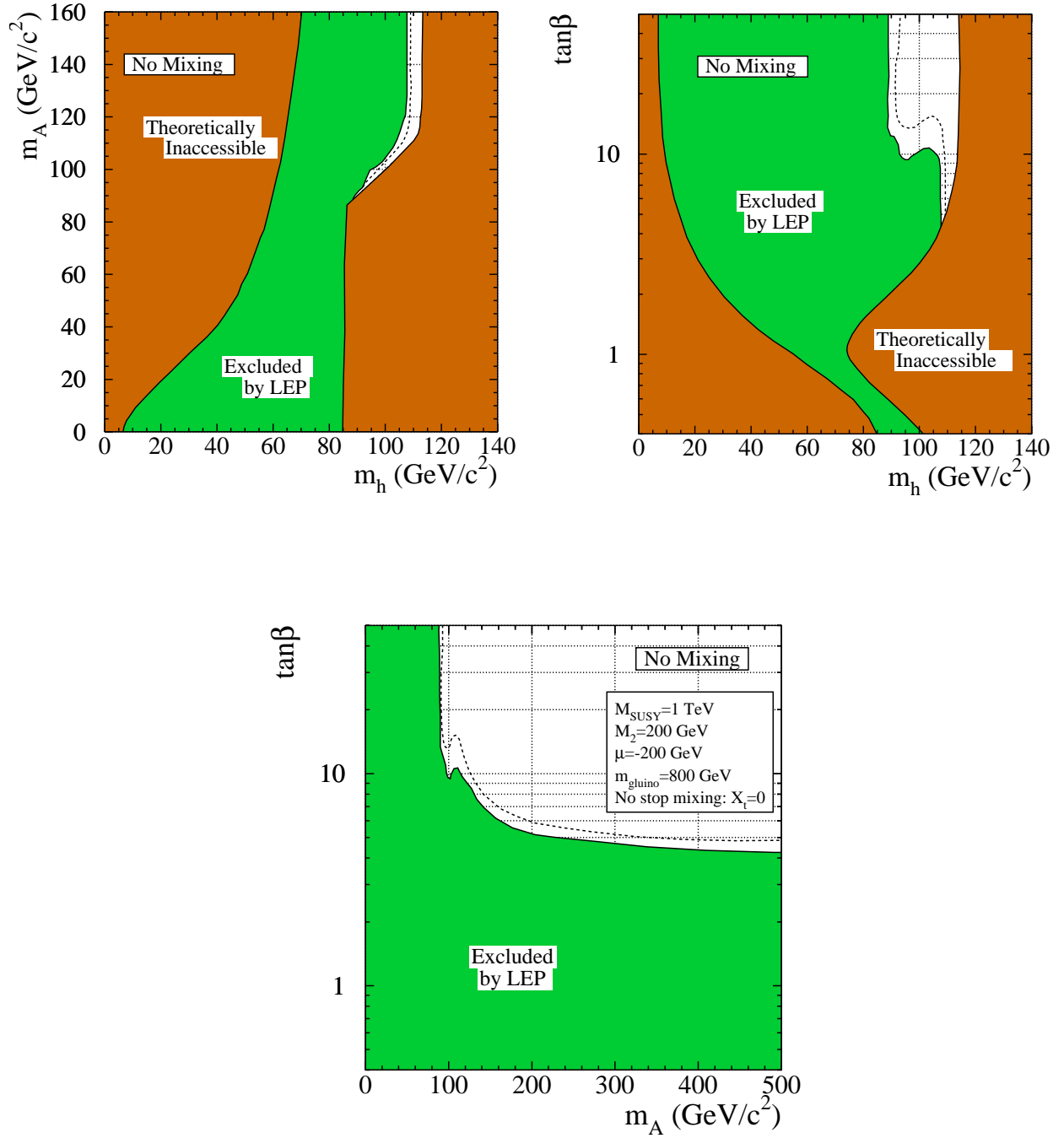


Figure 54: Excluded regions for the neutral Higgs bosons search in the MSSM in the no-mixing case

However, for a heavy pseudoscalar boson A the second process is decoupled and one basically has the same production rate as in the SM. Therefore, in this case the SM experimental limit is applicable also to the MSSM.

To present the result for the Higgs search in the MSSM, various variables can be used. The most popular ones are (m_h, m_A) , $(m_h, \tan\beta)$ and $(m_A, \tan\beta)$ planes. They are shown below in

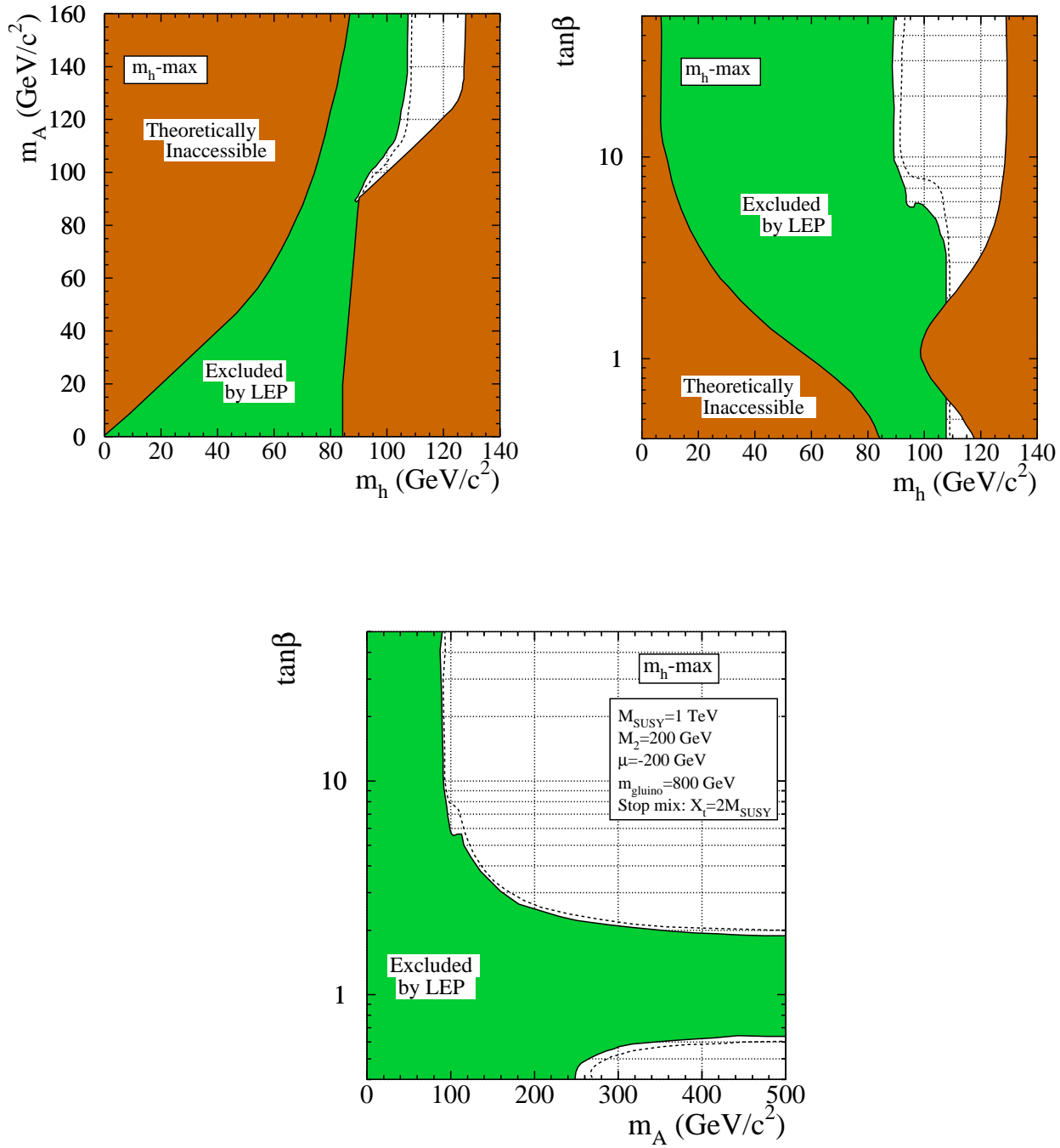


Figure 55: Excluded regions for the neutral Higgs bosons search in the MSSM in the maximal mixing case

Figs.54-55 for two particular cases: no-mixing and maximal mixing in the stop sector [66]. For comparison the theoretically allowed regions are shown. One can see that

- a) low $\tan\beta$ solution ($0.5 < \tan\beta < 3.3$) is already excluded;
- b) very small region for the lightest neutral Higgs boson mass is left (specially for the no-mixing case).

As it has been explained, in the MSSM one has also the charged Higgs bosons. The searches for the charged Higgs bosons are the attempts to look beyond the Standard Model. It is basically the same in the MSSM and in any two Higgs doublet model. The charged Higgs bosons are produced in pairs in an annihilation process like any charged particles. The couplings are the standard EW couplings and the only unknown quantity is the charged Higgs mass. However, the branching ratios for the decay channels depend on the mass and the model. A large background comes from the W -pair production. Nonobservation of charged Higgs bosons at LEP gives the lower limit on their masses. The combined exclusion plot for various channels is shown in Fig.56. This imposes the absolute lower limit on the charged Higgs boson mass [69]

$$m_{H^\pm} > 77.5 \text{ GeV}/c^2 \quad @ \quad 95\% \text{ C.L.} \quad (7.13)$$

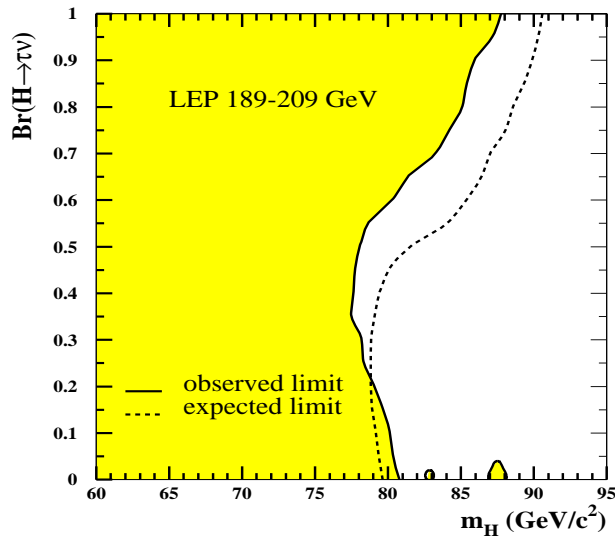


Figure 56: Combined exclusion plot for the charged Higgs boson

Tevatron and LHC

With the LEP shut down, further attempts to discover the Higgs boson are connected with the Tevatron and LHC hadron colliders.

Tevatron will start the Run II next year and will reach the c.m. energy of 2 TeV with almost 10 times greater luminosity than in RUN I. However, since it is a hadron collider, not the full energy goes into collision taken away by those quarks in a proton that do not take part in the interaction. Having a very severe background, this collider needs a long time of running to reach the integrated luminosity required for the Higgs discovery. A combined CDF/D0 plot [78] shows the integrated luminosity at Tevatron as a function of the Higgs mass (see Fig.57). The three curves correspond to 2σ (95% confidence level), 3σ and 5σ signal necessary for exclusion, evidence and discovery of the Higgs boson, respectively. One can see that the integrated luminosity of 2 fb^{-1} , which is planned to be achieved at the end of 2001, will allow one to exclude the Higgs boson with the mass of an order of 115 GeV, i.e., just the limit reached by LEP. One will need RUN III to reach 10 fb^{-1} to cover the most interesting interval, even at the level of exclusion (2σ).

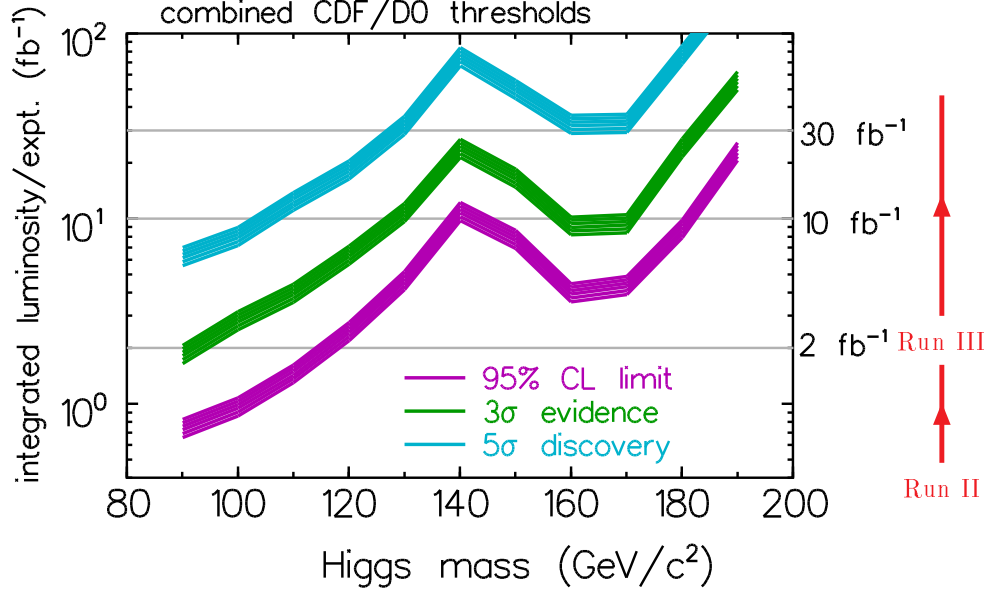


Figure 57: Integrated luminosity needed for exclusion (2σ), evidence (3σ) and discovery (5σ) of the Higgs boson at Tevatron

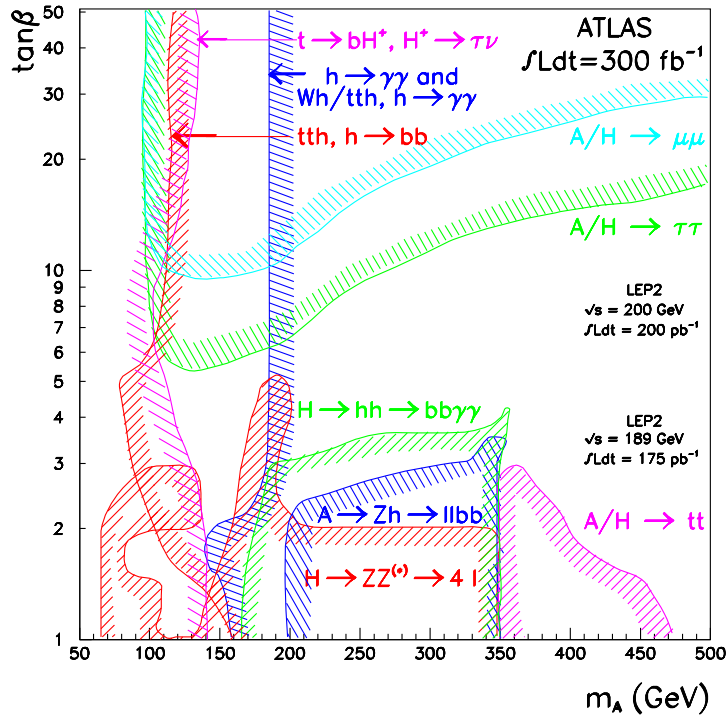


Figure 58: Exclusion plots for LHC hadron collider for different Higgs decay modes

To find the Higgs boson, one will need still greater integrated luminosity. The signatures of the Higgs boson are related to the dominant decay modes which depend on the mass of the

Higgs boson. In the Tevatron region they are

$$\begin{aligned} H &\rightarrow b\bar{b}, & 100 < m_H < 140 \text{ GeV}, \\ H &\rightarrow WW^*, & 140 < m_H < 175 \text{ GeV}, \\ H &\rightarrow ZZ^*, & 175 < m_H < 190 \text{ GeV}. \end{aligned} \tag{7.14}$$

The LHC hadron collider is the ultimate machine for a new physics at the TeV scale. Its c.m. energy is planned to be 14 TeV with very high luminosity up to a few hundred fb^{-1} . It is supposed to start operating in 2006. In principle, LHC will be able to cover the whole interval of SUSY and Higgs masses up to a few TeV. It will either discover the SM or the MSSM Higgs boson, or prove their absence. In terms of exclusion plots shown in Figs.54, 55 the LHC collider will cover the whole region [79]. Various decay modes allow one to probe different areas, as shown in Fig.58, though the background will be very essential.

8 Conclusion

LEP II has neither discovered the new physics, nor has proven the existence of the Higgs boson. However, it gave us some indication that both of them exist. Supersymmetry is now the most popular extension of the Standard Model. It promises us that new physics is round the corner at a TeV scale to be exploited at new machines of this decade. If our expectations are correct, very soon we will face new discoveries, the whole world of supersymmetric particles will show up and the table of fundamental particles will be enlarged in increasing rate. If we are lucky, probably we will soon have the table of sparticles in new addition of Sparticle Data Group (see Fig.59) [80]. This would be a great step in understanding the microworld. If not, still new discoveries are in agenda.

Acknowledgements

I am grateful to the organizers of the School at Caramulo for providing very nice atmosphere during the school and to the students for their interest, patience and attention. I would like to thank V.Velizhanin for his help in preparation of this manuscript. Financial support from RFBR grants # 99-02-16650, # 98-02-17453 and # 00-15-96691 and Heienberg-Landau Programme is kindly acknowledged.



The [SPDG is an international collaboration](#) that reviews Sparticle Physics and related areas of Astrophysics, and compiles/analyzes data on particle properties. SPDG products are distributed to 130,000 physicists, teachers, and other interested people. The **Review of Sparticle Physics** is the most cited publication in particle physics during the last twenty years. Plots of [SPDG statistics](#) are available.

Mirror sites: [USA \(LBNL\)](#) [Brazil](#) [CERN](#) [Italy \(Genova\)](#) [Japan \(KEK\)](#) [Russia \(Novosibirsk\)](#) [Russia \(Protvino\)](#) [UK \(Durham\)](#)

[Review of Sparticle Physics](#) [Charts, Educational materials, Sparticle Adventure](#) [Information and Databases](#)
[US-HEPFOLK](#) [Sparticle Physics: Twenty Years of Discoveries](#) [Home Pages of major HEP labs](#)

The Review of Sparticle Physics

[C. Caso *et al.*](#) The European Physical Journal **C103** (2018) 1 ([2018 Authors](#))

- **2019** [2019 Web update of Reviews, Tables, Plots](#) New November 2, 2019
- **2019** [2019 Web update of Sparticle Listings](#) New July 6, 2019
- **2018** [2018 Summary Tables and Conservation Laws](#)
- [2018 Reviews, Tables, Plots \(incl. Intro. Text\)](#) Superseded by [2019 Web Version](#)
- [2018 Sparticle Listings \(published version\)](#) Superseded by [2019 Web Version](#)
- [Errata](#) (last changed January 18, 2020)
- Archived WWW editions: [2017](#) [2016](#) [2015](#)
- [Descriptions](#) of the Summary Tables, Reviews, Listings, etc.
- [Ordering information](#) and list of products
- [2018 Authors](#) and [Directory of Sparticle Data Group Authors, Associates, and Advisors](#)
- [Computer-readable files](#) – masses, widths, cross-sections, etc., including [Palm Pilot XXII](#) files.
- [Encoder tools](#) (for SPDG collaborators)

Figure 59: Foreseeable future: SParticle Data Group

References

- [1] D. E. Groom *et al.*, “Review of Particle Physics”, Eur. Phys. J. **C15** (2000) 1.
- [2] LEP EWWG, <http://lepewwg.web.cern.ch/LEPEWWG/plots/summer2000/>
- [3] Y. A. Golfand and E. P. Likhtman, *JETP Letters* **13** (1971) 452; D. V. Volkov and V. P. Akulov, *JETP Letters* **16** (1972) 621; J. Wess and B. Zumino, *Phys. Lett.* **B49** (1974) 52.
- [4] P. Fayet and S. Ferrara, *Phys. Rep.* **32** (1977) 249; M. F. Sohnius, *Phys. Rep.* **128** (1985) 41; H. P. Nilles, *Phys. Rep.* **110** (1984) 1; H. E. Haber and G. L. Kane, *Phys. Rep.* **117** (1985) 75; A. B. Lahanas and D. V. Nanopoulos, *Phys. Rep.* **145** (1987) 1.
- [5] J. Wess and J. Bagger, “*Supersymmetry and Supergravity*”, Princeton Univ. Press, 1983.
- [6] P. West, “*Introduction to Supersymmetry and Supergravity*”, World Scientific, 1986.
- [7] S. J. Gates, M. Grisaru, M. Roček and W. Siegel, “*Superspace or One Thousand and One Lessons in Supersymmetry*”, Benjamin & Cummings, 1983.
- [8] S. Weinberg, “*The quantum theory of fields. Vol. 3: Supersymmetry*”, Cambridge, UK: Univ. Press, 2000.
- [9] S. Coleman and J. Mandula, *Phys. Rev.* **159** (1967) 1251.
- [10] P. Nath and R. Arnowitt, *Phys. Lett.* **B56** (1975) 177; D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, *Phys. Rev.* **D13** (1976) 3214; S. Deser and B. Zumino, *Phys. Lett.* **B62** (1976) 335; see also “*Supersymmetry*”, S. Ferrara, ed. (North Holland/World Scientific, Amsterdam/Singapore, 1987) and Refs.[5]-[8].
- [11] G. G. Ross, “*Grand Unified Theories*”, Benjamin & Cummings, 1985.
- [12] S. Bethke, *J. Phys.* **G26** (2000) R27; hep-ex/0004021.
- [13] G. 't Hooft, *Nucl. Phys.* **B61**, (1973) 455;
W. A. Bardeen, A. Buras, D. Duke and T. Muta, *Phys. Rev.* **D 18**, (1978) 3998.
- [14] A. D. Martin, J. Outhwaite and M. G. Ryskin, *Phys. Lett.* **B492** (2000) 69; hep-ph/0008078.
- [15] U. Amaldi, W. de Boer and H. Fürstenau, *Phys. Lett.* **B260** (1991) 447.
- [16] I. Antoniadis, C. Kounnas, and K. Tamvakis, *Phys. Lett.* **119B** (1982) 377.
- [17] M. B. Green, J. H. Schwarz and E. Witten, “*Superstring Theory*”, Cambridge, UK: Univ. Press, 1987. *Cambridge Monographs On Mathematical Physics*.
- [18] A. M. Polyakov, “*Gauge Fields And Strings*”, CHUR, Switzerland: Harwood, 1987, *Contemporary Concepts in Physics*, 3.
- [19] J. Wess and B. Zumino, *Nucl. Phys.* **B78** (1974) 1.
- [20] F. A. Berezin, “*The Method of Second Quantization*”, Moscow, Nauka, 1965.
- [21] K. Huang, “*Quarks, Leptons And Gauge Fields*”, Singapore, World Scientific, 1982.
- [22] P. Fayet and J. Illiopoulos, *Phys. Lett.* **B51** (1974) 461.

- [23] L. O’Raifeartaigh, *Nucl. Phys.* **B96** (1975) 331.
- [24] R. Barbieri, *Riv. Nuo. Cim.* **11** (1988) 1;
H. E. Haber, ”*Introductory Low-Energy Supersymmetry*”, Lectures given at TASI 1992, (SCIPP 92/33, 1993), hep-ph/9306207.
W. de Boer, ”*Grand Unified Theories and Supersymmetry in Particle Physics and Cosmology*”, *Progr. in Nucl. and Particle Phys.*, **33** (1994) 201, hep-ph/9402266;
D. I. Kazakov, ”*Mimimal Supersymmetric Extension of the Standard Model*”, *Surveys in High Energy Physics*, **10** (1997) 153, wwwtheor.itep.ru/school96/
D. I. Kazakov, ”*Supersymmetry in Particle Physics: Renormalization Group Viewpoint*”, Review talk at the Conf. RG-2000, hep-ph/0001257.
- [25] see e.g. H. Haber in [24].
- [26] <http://atlasinfo.cern.ch/Atlas/documentation/EDUC/physics14.html>
- [27] P. Fayet, *Phys. Lett.* **B69** (1977) 489; G. Farrar and P. Fayet, *Phys. Lett.* **B76** (1978) 575.
- [28] P. Fayet, *Nucl. Phys.* **B90**(1975) 104; A. Salam and J. Srahddee, *Nucl. Phys.* **B87**(1975) 85.
- [29] H. Dreiner and G. G. Ross, *Nucl. Phys.* **B365** (1991) 597, K. Enqvist, A. Masiero and A. Riotto, *Nucl. Phys.* **B373** (1992) 95,
H. Dreiner and P. Morawitz, *Nucl. Phys.* **B428** (1994) 31; H. Dreiner and H. Pois, preprint NSF-ITP-95-155; hep-ph/9511444,
V. Barger, M. S. Berger , R. J. N. Philips and T. Wöhrmann, *Phys. Rev.* **D53** (1996) 6407.
- [30] L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27** (1983) 2359; S. K. Soni and H. A. Weldon, *Phys. Lett.* **B126** (1983) 215; I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* **B256** (1985) 557.
- [31] H. P. Nilles, *Phys. Lett.* **B115** (1982) 193; A. H. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49** (1982) 970; *Nucl. Phys.* **B227** (1983) 121; R. Barbieri, S. Ferrara and C. A. Savoy, *Phys. Lett.* **B119** (1982) 343; E. Cremmer, P. Fayet and L. Girardello, *Phys. Lett.* **B122** (1983) 41; L. Ibáñez, *Phys. Lett.* **B118** (1982) 73; H. P. Nilles, M. Srednicki and D. Wyler, *Phys. Lett.* **B120** (1983) 346.
- [32] M. Dine and A. E. Nelson, *Phys. Rev.* **D48** (1993) 1277, M. Dine, A. E. Nelson and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362, hep-ph/9408384; M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, *Phys. Rev.* **D53** (1996) 2658, hep-ph/9507378.
- [33] L. Randall and R. Sundrum, *Nucl. Phys.* **B557** (1999) 79, hep-th/9810155; G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, *JHEP*, **9812** (1998) 027, hep-ph/9810442.
- [34] D. E. Kaplan, G. D. Kribs and M. Schmaltz, *Phys. Rev.* **D62** (2000) 035010, hep-ph/9911293; Z. Chacko, M. A. Luty, A. E. Nelson and E. Ponton, *JHEP*, **0001** (2000) 003, hep-ph/9911323.
- [35] L. Girardello and M. Grisaru, *Nucl. Phys.* **B194** (1982) 65.
- [36] M. E. Peskin, ”*Theoretical summary lecture for EPS HEP99*”, hep-ph/0002041, see also Ref.[80].
- [37] N. Polonsky and A. Pomarol, *Phys. Rev. Lett.* **73** (1994) 2292; *Phys. Rev.* **D51** (1995) 6532.

- [38] G. G. Ross and R. G. Roberts, *Nucl. Phys.* **B377** (1992) 571.
V. Barger, M. S. Berger and P. Ohmann, *Phys. Rev.* **D47** (1993) 1093.
- [39] W. de Boer, R. Ehret and D. Kazakov, *Z. Phys.* **C67** (1995) 647;
W. de Boer et al., *Z. Phys.* **C71** (1996) 415.
- [40] N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, *Nucl. Phys.* **B158** (1979) 295;
M. Lindner, *Z. Phys.* **C31** (1986) 295; M. Sher, *Phys. Rev.* **D179** (1989) 273;
M. Lindner, M. Sher and H. W. Zaglauer, *Phys. Lett.* **B228** (1989) 139.
- [41] L. E. Ibáñez, C. Lopéz and C. Muñoz, *Nucl. Phys.* **B256** (1985) 218.
- [42] W. Barger, M. Berger, P. Ohman, *Phys. Rev.* **D49** (1994) 4908.
- [43] G. Brooijmans [CDF and D0 Collaborations], hep-ex/0005030.
<http://www-d0.fnal.gov/public/top/top>
<http://www-cdf.fnal.gov/physics/new/top/results/mass/combine>
- [44] V. Barger, M. S. Berger, P. Ohmann and R. J. N. Phillips, *Phys. Lett.* **B314** (1993) 351.
- [45] P. Langacker and N. Polonsky, *Phys. Rev.* **D49** (1994) 1454.
- [46] S. Kelley, J. L. Lopez and D.V. Nanopoulos, *Phys. Lett.* **B274** (1992) 387.
- [47] P. H. Chankowski, S. Pokorski and J. Rosiek, *Nucl. Phys.* **B423** (1994) 437;
- [48] M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, *Nucl. Phys.* **B426** (1994) 269.
- [49] H. Arason et al., *Phys. Rev. Lett.* **67** (1991) 2933.
- [50] R. Poling (University of Minnesota), Talk at the Lepton - Photon '99 Conference, Stanford University, USA, 9-14 August 1999; see also CLEO Collaboration, CLEO CONF 98-17, ICHEP98 1011.
- [51] R. Barate et al. (ALEPH Collaboration), *Phys. Lett.* **B429** (1998) 169.
- [52] F. Borzumati, *Z. Phys.* **C63** (1994) 291.
- [53] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, *Nucl. Phys.* **B353** (1991) 591 and references therein; N. Oshimo, *Nucl. Phys.* **B404** (1993) 20.
- [54] ALEPH Collaboration, "Search for supersymmetric particles in $e^+ e^-$ collisions at \sqrt{s} up to 202-GeV and mass limit for the lightest neutralino", hep-ex/0011047.
- [55] S. Abel et al. [SUGRA Working Group Collaboration], *Report of the SUGRA working group for run II of the Tevatron*, hep-ph/0003154.
D0 Coll., "Search for Squarks and Gluinos in Events Containing Jets and a Large Imbalance in Transverse Energy", *Phys. Rev. Lett.* **83** (1999) 4937.
- [56] G. Börner, "The early Universe", Springer Verlag, 1991.
- [57] E. W. Kolb and M. S. Turner, "The early Universe", Addison-Wesley, 1990.
- [58] G. Steigman, K. A. Olive, D. N. Schramm, M. S. Turner, *Phys. Lett.* **B176** (1986) 33;
J. Ellis, K. Enquist, D.V. Nanopoulos, S. Sarkar, *Phys. Lett.* **B167** (1986) 457;
G. Gelmini and P. Gondolo, *Nucl. Phys.* **B360** (1991) 145.

- [59] M. Drees and M. M. Nojiri, *Phys. Rev.* **D47** (1993) 376;
J. L. Lopez, D. V. Nanopoulos and H. Pois, *Phys. Rev.* **D47** (1993) 2468;
P. Nath and R. Arnowitt, *Phys. Rev. Lett.* **70** (1993) 3696;
J. L. Lopez, D. V. Nanopoulos and K. Yuan, *Phys. Rev.* **D48** (1993) 2766.
- [60] G. L. Kane, C. Kolda, L. Roszkowski and J. D. Wells, *Phys. Rev.* **D49** (1994) 6173.
- [61] L. Roszkowski, *Univ. of Michigan Preprint*, UM-TH-93-06; *Phys. Rev.* **D50** (1994) 4842.
- [62] F. M. Borzumati, M. Olechowski and S. Pokorski, *Phys. Lett.* **B349** (1995) 311.
- [63] W.de Boer, H.-J.Grimm, A.V.Gladyshev and D.I.Kazakov, *Phys. Lett.* **B438** (1998) 281.
- [64] W. de Boer, M. Huber, A. V. Gladyshev and D. I. Kazakov, hep-ph/0007078.
- [65] J. Gunion, H. Haber, G. Kane and S. Dawson, *"Higgs Hunter's Guide"*, Addison-Wesley, New York, 1990.
- [66] The LEP Higgs Working group, R. Bock et al., CERN-EP-2000-055 and LEP experiments, ALEPH 2000-28, DELPHI 2000-050, L3-Note 2525, OPAL TN646.
- [67] M. Sher, *Phys. Lett.* **B317** (1993) 159; C. Ford, D. R. T. Jones, P. W. Stephenson and M. B. Einhorn, *Nucl. Phys.* **B395** (1993) 17; G. Altarelli and I. Isidori, *Phys. Lett.* **B337** (1994) 141; J. A. Casas, J. R. Espinosa and M. Quiros, *Phys. Lett.* **B342** (1995) 171.
- [68] T. Hambye, K. Reisselmann, *Phys. Rev.* **D55** (1997) 7255; H. Dreiner, hep-ph/9902347.
- [69] P. Bock *et al.* [ALEPH, DELPHI, L3 and OPAL Collaborations], CERN-EP-2000-055; http://lephiggs.web.cern.ch/LEPHIGGS/papers/osaka_note.ps
- [70] R. Barate *et al.* [ALEPH Collaboration], *"Observation of an excess in the search for the standard model Higgs boson at ALEPH,"* hep-ex/0011045.
- [71] M. Acciarri *et al.* [L3 Collaboration], *"Higgs candidates in e^+e^- interactions at $\sqrt{s} = 206.6$ GeV,"* hep-ex/0011043.
- [72] J. Ellis, G. Ridolfi, F. Zwirner, *Phys. Lett.* **B262** (1991) 477; A. Brignole, J. Ellis, G. Ridolfi, F. Zwirner, *Phys. Lett.* **B271** (1991) 123.
- [73] M. Carena, J. R. Espinosa, M. Quiros and C. E. M. Wagner, *Phys. Lett.* **B355** (1995) 209; J. Ellis, G. L. Fogli and E. Lisi, *Phys. Lett.* **B333** (1994) 118.
- [74] A.V.Gladyshev, D.I.Kazakov, W.de Boer, G.Burkart, R.Ehret, *Nucl. Phys.* **B498** (1997) 3.
- [75] M. Carena, M. Quiros and C. E. M. Wagner, *Nucl. Phys.* **B461** (1996) 407.
- [76] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Lett.* **B455** (1999) 179, hep-ph/9903404; *Eur. Phys. J.* **C9** (1999) 343, hep-ph/9812472.
- [77] M. Masip, R. Muñoz-Tapia and A. Pomarol, *Phys. Rev.* **D57** (1998) 5340.
- [78] M. Carena *et al.*, *"Report of the Tevatron Higgs working group"*, hep-ph/0010338.
- [79] ATLAS TDR, <http://www.cern.ch/ATLAS/GROUPS/PHYSICS/TDR>.
- [80] J. Bagger, summary talk at SUSY2K
http://wwwth.cern.ch/susy2k/susy2ktalks/bagger_plenary.pdf.gz

DELPHI $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ limits at 202 GeV

